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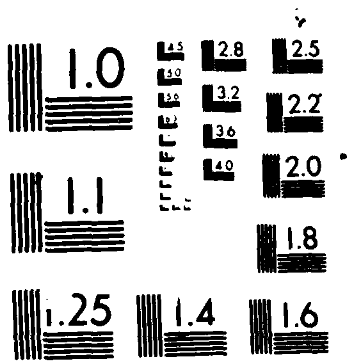

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CHARACTERIZATION OF STRIPLINE CROSSING BY TRANSVERSE RESONANCE METHOD

TECHNICAL REPORT

TOMOKI UWANO AND TATSUO ITOH

JANUARY 1987

OFFICE OF NAVAL RESEARCH

CONTRACT NO. N00014-79-C-0553

UNIVERSITY OF TEXAS
DEPARTMENT OF ELECTRICAL ENGINEERING
AUSTIN, TEXAS, 78712

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Abstract

A method of analysis is described for characterizing the discontinuities made of orthogonally crossed two striplines on a suspended structure. The method of analysis is based on generalized transverse resonant technique extended here to 4-port configurations. The technique is used for determination of resonant structure at a given frequency and subsequently the equivalent circuit parameters of the discontinuities.

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1. INTRODUCTION OF THE PROBLEM

Stripline crossings of the multi-layer printed circuitboard are commonly used in digital circuit design. As the signal frequency gets higher due to high speed processors, an accurate wave analysis of the characteristics of the crossing becomes important. In addition, the crossing of strips on both sides of the suspended substrate often appears in microwave and millimeter wave integrated circuits [1]. To date little has been reported on the exact analysis of such structures.

The problem presented here is to characterize the discontinuities of orthogonally crossed two striplines. The structure to be analyzed is shown schematically in Fig.1-1 along with the coordinate system. Two striplines are crossed orthogonally on opposite sides of the substrate. Auxiliary conducting planes are added to convert the structure to a closed one. Depending on which ground plane (at $z = -h1$ or $z = c$) is associated with each strip, the structure may be regarded as consisting of either inverted or suspended striplines.

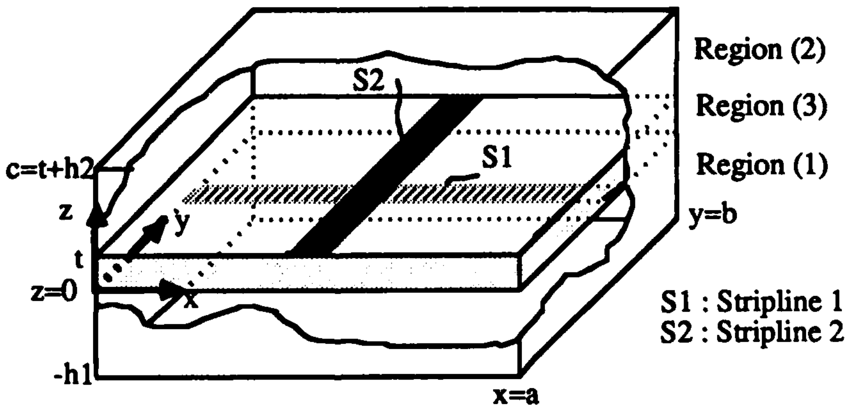


Fig. 1-1 Structure for the problem

It is assumed that each pair of opposing side walls does not influence electromagnetic fields guided by the strip parallel to the walls but only the field guided along the orthogonal directions. This assumption is valid as long as the field remains confined in the proximity of the two striplines. Thus, surface wave and radiation phenomena are excluded. For the sake of generality, two different dielectric layers with dielectric constant $\varepsilon 1$ and $\varepsilon 2$ are assumed in the region (1) and (2), respectively. The auxiliary walls are used for field analysis purpose. They permit the structure to be analyzed as a rectangular waveguide discontinuity problem.

The method for analysis is based on a generalized "transverse resonance technique" introduced for finline step discontinuity problem [2]. The technique is extended here to a 4-port configuration treated in this report. The method consists of two parts. First, the resonant structure created by auxiliary walls is described in terms of network representation containing a reactive 4-port. For a fixed resonant frequency, we try to find as many resonator sizes as required for extraction of 4-port matrix elements. The second part of analysis is a full-wave electromagmetic field analysis in which the resonant frequency is found as an eigenvalue problem. A unique feature of this second part of the analysis is to view the problem as the waveguide scattering for the waves traveling in the direction normal to the subatrate surface.

2. CIRCUIT REPRESENTATION

2-1. Resonance Method

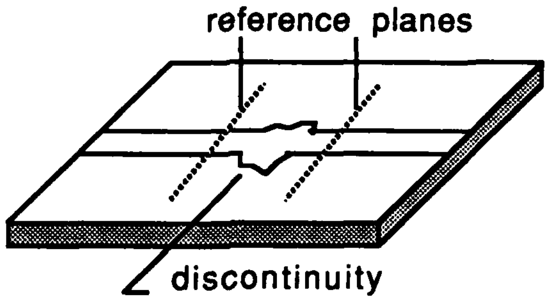


Fig. 2-1 Microstrip line with some discontinuities

In this section, the resonance method applied to 2-port network [3] is briefly described. Suppose we have a transmission line with some discontinuities as shown in Fig. 2-1. With appropriate reference planes for the input - output ports, the discontinuities can be represented as a 2-port network matrix form :

$$\begin{bmatrix} V1 \\ V2 \end{bmatrix} = [Z] \begin{bmatrix} I1 \\ I2 \end{bmatrix}, \quad [Z] = \begin{bmatrix} Z11 & Z12 \\ Z21 & Z22 \end{bmatrix} \tag{2-1}$$

where Z11, Z12, Z21, Z22 are unkown parameters. In the assumption of lossless and reciprocal network, we have only three real unknowns in the matrix. If the two ports are reactively terminated, the entire circuit becomes a resonator. This is shown in Fig. 2-2 where Zoi (i = 1,2) is characteristic impedance, βi is the propagation constant and li is the length of the i th transmission line. In Fig. 2-2, the resonance condition is expressed as

$$(Z11 + Z1)(Z22 + Z2) - Z12^2 = 0 \quad . \tag{2-2}$$

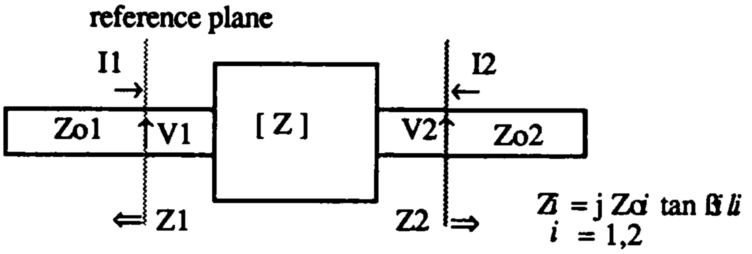


Fig. 2-2 Entire circuit as a resonator for a 2-port network

If the same resonant frequency is obtained with three different pairs of Z1 and Z2, Eq. (2-2) yields three different equations and the Z parameters can be solved. The reference plane can be placed at any position as long as it is on the continuous part of the transmission line and the field disturbance due to the discontinuities remains the same.

The equivalent circuit of a 2-port network can be commonly expressed as shown in Fig. 2-3.

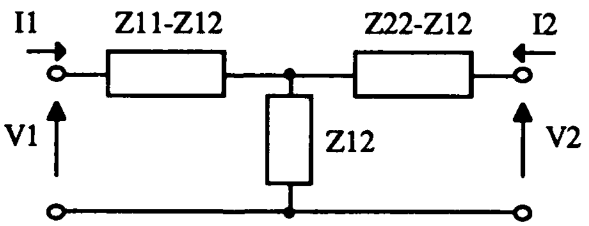


Fig. 2-3 Equivalent circuit of a 2-port network

Since the value of Z12 is solved with its square value in Eq. (2-2), the sign of the value must be chosen so that the equivalent circuit has physically proper frequency performance in the regarding frequency region. If the reference planes are placed close to the discontinuities, where the fields are much influenced by them, negative capacitors or inductors may appear in an equivalent circuit representation.

2-2. 4-Port Network

In this section, a procedure for a 2-port resonance method is extended to a 4-port configuration. The crossing between the two suspended striplines can be represented as a 4-port network at some reference planes sufficiently far from the discontinuity region. Each port is terminated with a reactance corresponding to the line section between the reference plane and the auxiliary wall as shown in Fig. 2-4.

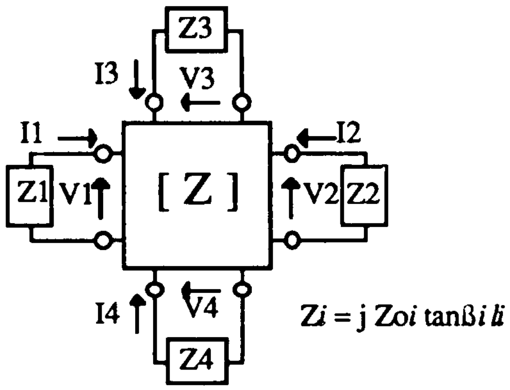


Fig. 2-4 4-port network for the problem

The network equations for the entire circuit is expressed in matrix form as

$$[\,V\,] = [\,Z\,]\,[\,I\,] \tag{2-3a}$$

$$[\,V\,] = -\,\mathrm{diag}\,[\,Z_i\,]\,[\,I\,] \tag{2-3b}$$

thus

$$[\,[\,Z\,] + \mathrm{diag}[\,Z_i\,]\,]\,[\,I\,] = 0 \tag{2-3c}$$

where Z_i (i = 1,2,3,4) are the terminal impedance : $Z_i = j\,Z_{oi} \tan \beta_i l_i$. The entire structure is assumed to be lossless so that the Z parameters of the 4-port network are imaginary and the resonant frequency is real. The resonant frequency is obtained from the condition that the voltages and currents are non-trivial in the absence of sources. From Eq. (2-3c), this condition is

$$\det \|\,[\,Z\,] + \mathrm{diag}\,[\,Z_i\,]\,\| = 0 \tag{2-4}$$

The values of Z_i's can be specified once the distance to the wall is fixed while the Z parameters are to be determined. The impedance matrix of a reciprocal 4-port lossless

network possesses in general 10 independent imaginary parameters. In the present case, however, because of the symmetry of the structure, only five parameters are needed to characterize the Z matrix :

$$\begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 & Z13 & Z13 \\ Z12 & Z11 & Z13 & Z13 \\ Z13 & Z13 & Z33 & Z34 \\ Z13 & Z13 & Z34 & Z33 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ I4 \end{bmatrix} \qquad (2\text{-}5)$$

From Eq. (2-5), the resonant condition of Eq. (2-4) can be written in the following form :

$$\{ (Z11+Z1)(Z11+Z2)-Z12^2 \} \{ (Z33+Z3)(Z33+Z4)-Z34^2 \}$$
$$- 4\, Z13^2 \{ Z11-Z12+(Z1+Z2)/2 \} \{ Z33-Z34+(Z3+Z4)/2 \} = 0 \qquad (2\text{-}6)$$

The derivation of the equation is detailed in Appendix A. It is observed that, in the limit as Z13 equals zero, Eq. (2-6) reduces to

$$(Z11 + Z1) (Z11 + Z2) - Z12^2 = 0 \quad \text{or} \qquad (2\text{-}7a)$$
$$(Z33 + Z3) (Z33 + Z4) - Z34^2 = 0 \quad . \qquad (2\text{-}7b)$$

As expected, when the two striplines are uncoupled, the resonant condition splits into those of the individual lines. The unknown Z parameters as well as terminal impedances *Zi*'s are frequency dependent. Eq. (2-6) can be regarded as a function of ω equated to zero. The Z parameters in Eq. (2-6) may be solved if the five different equations are derived corresponding to five different set of *Zi*'s at the same resonant frequency.

Now we can show that by properly choosing the terminal impedance *Zi*'s, the resonant conditions are simplified so that the problem is solved analytically. First, let us choose the terminal impedances in a symmetrical way, i.e., $l1=l2$ and $l3=l4$ so that

$$Z1 = Z2 \qquad (2\text{-}8a)$$
$$Z3 = Z4 \quad . \qquad (2\text{-}8b)$$

The above conditions correspond to the side walls located symmetrically with respect to the discontinuities. If Eqs. (2-8) are applied, Eq. (2-6) can be factorized in the form

$$(Z11+Z1-Z12)\,(Z33+Z3-Z34)\,\{(Z11+Z12+Z1)(Z33+Z34+Z3)-4\,Z13^2\} = 0\,. \quad (2\text{-}9a)$$

Thus

$$Z11 + Z1 - Z12 = 0 \quad \text{or} \quad (2\text{-}9b)$$

$$Z33 + Z3 - Z34 = 0 \quad \text{or} \quad (2\text{-}9c)$$

$$(Z11 + Z12 + Z1)\,(Z33 + Z34 + Z3) - 4\,Z13^2 = 0\,. \quad (2\text{-}9d)$$

With each factor in Eq. (2-9a) equated to zero, the eigenvalues for the matrix in Eq. (2-3c) are obtained ; each eigenvalue ω is then the resonant frequency under the condition of the corresponding eigenvector of the currents. When the first factor in Eq. (2-9a) is equated to zero, i.e. Eq.(2-9b) is satisfied, the eigenvector of Eq. (2-4) is easily found as detailed in Appendix B :

$$I1 = -I2 \quad (2\text{-}10a)$$

$$I3 = I4 = 0\,. \quad (2\text{-}10b)$$

This condition corresponds to an odd resonance of the structure shown in Fig. 2-5(a). The structure behaves as if an electric wall is placed symmetrically along at the center of stripline 2. For the given resonant frequency, the required resonance condition provides the quantity Z11-Z12 for a given value of Z1. I3 = I4 = 0 (therefore V3 = V4 = 0 unless Zi is infinite) does not imply that the electromagnetic field is zero along the strip 2, but only that the electromagnetic field of the dominant mode is zero ; it is anticipated that the field is localized at the crossing region along the strip 2 as illustrated in Fig. 2-6(b).

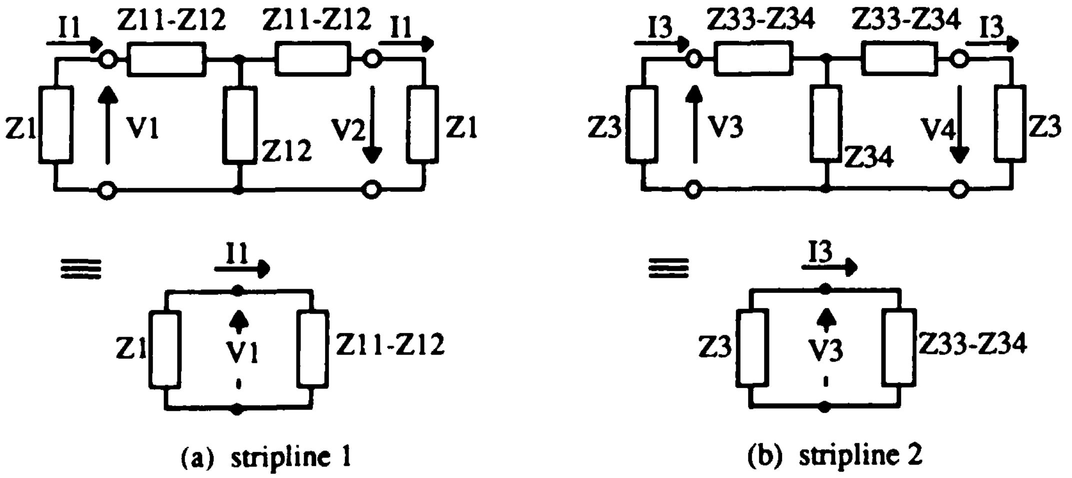
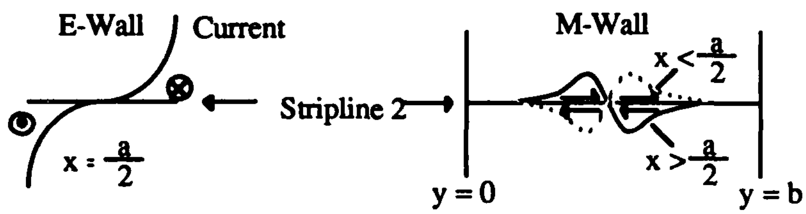


Fig. 2-5 Equivalent circuits for an odd resonance



(a) Cross sectional view (b) Side view

Fig. 2-6 Longitudinal current flow on stripline 2 with E-wall

Similary, when Eq. (2-9c) is satisfied, an odd resonance of stripline 2 is obtained

$$I1 = I2 = 0 \tag{2-11a}$$

$$I3 = -I4 \tag{2-11b}$$

The resonant circuit is expressed in Fig. 2-5(b). It is interesting to note that the individual parameters of the pairs of ports 1-2 and 3-4 are uncoupled in the odd resonance modes. Finally, from Eq. (2-9d), the eigenvector for an even resonance is obtained :

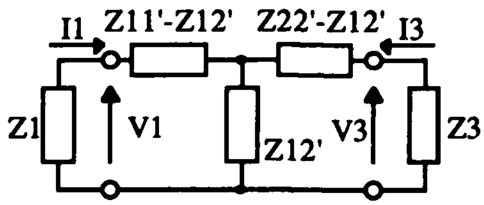
$$I1 = I2 \tag{2-12a}$$

$$I3 = I4 \tag{2-12b}$$

Substitution of this condition into Eq. (2-5) yields the 2-port network matrix equation

$$\begin{bmatrix} V1 \\ V3 \end{bmatrix} = \begin{bmatrix} Z11+Z12 & 2\,Z13 \\ 2\,Z13 & Z33+Z34 \end{bmatrix} \begin{bmatrix} I1 \\ I3 \end{bmatrix} \tag{2-13}$$

whose equivalent circuit is illustrated in Fig. 2-7 together with terminal impedances.



Z11' = Z11 + Z12
Z12' = 2 Z13
Z22' = Z33 + Z34

Fig. 2-7 Equivalent circuit for an even resonance

From the condition of Eqs. (2-12), it is postulated that the even resonance structure has two magnetic walls at each center of the striplines. The use of symmetry, therefore, has reduced the 4-port network problem to that of a 2-port. For a given resonant frequency, three different pairs of Z1 and Z3 (with Z1 = Z2 and Z3 = Z4) are used to compute three quantities Z11′,Z12′ and Z22′ which denote the elements of the matrix in Eq. (2-13) :

$$Z11' = Z11 + Z12 \qquad (2\text{-}14a)$$

$$Z22' = Z33 + Z34 \qquad (2\text{-}14b)$$

$$Z12' = 2\,Z13 \qquad (2\text{-}14c)$$

Combining the results with those for the two structures in Fig. 2-5, we obtain all five Z parameters. For the procedure illustrated above, we must know the propagation constants of the two isolated striplines, i. e., ß1 and ß3. The quantities are necessary to obtain Z1 and Z3 :

$$z1 = j \tan \beta 1(a - w2)/2 \qquad (2\text{-}15a)$$

$$z3 = j \tan \beta 3(b - w1)/2 \qquad (2\text{-}15b)$$

where the impedances are normalized. These expressions are obtained for a specific choice of the reference planes as shown in Fig. 2-8. The field analysis can be also used to determine ß1 and ß3. This can be done simply by setting ß1 = π/a and ß3 = π/b at a specified resonant frequency for the isolated stripline.

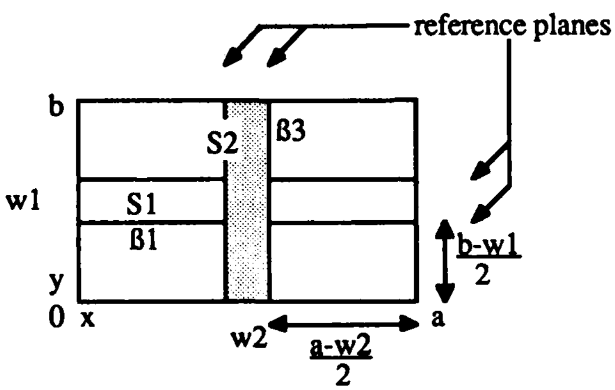


Fig. 2-8 A choice of reference planes (top view)

2-3. Equivalent Circuit

It is important to find an equivalent circuit representation of the 4-port structure so that it is most convenient for the present analysis. Intuitive insights are needed to figure out a good expression which well represents a structure in a broad frequency range. A basic idea for choosing proper elements is to use the knowlege of field distributions. If the E field originating from a stripline increases in the proximity of discontinuities, it may be represented by a parallel capacitive element. When the H field circling around a stripline changes, it may be represented by a series inductive element.

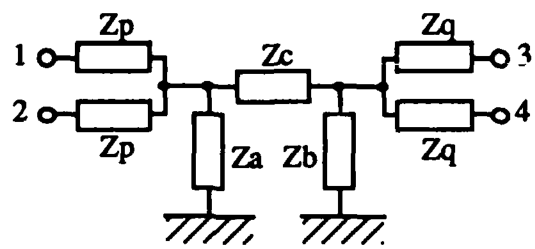


Fig. 2-9 Equivalent circuit for the problem

For the structure in Fig. 1-1, one possible equivalent circuit representation could be chosen as shown in Fig. 2-9. The circuit already takes the symmetry properties into account. Zc is used to represent the coupling capacitance between the two strips. Zp+Zp and Zq+Zq represent the inductances associated with the two stripline sections, while Za and Zb represent the two strip-to-ground capacitances. The Z matrix for the circuit in Fig. 2-9 is given as

$$\begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 & Z13 & Z13 \\ Z12 & Z11 & Z13 & Z13 \\ Z13 & Z13 & Z33 & Z34 \\ Z13 & Z13 & Z34 & Z33 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ I4 \end{bmatrix} \qquad (2\text{-}16a)$$

where

$$Z11 = Zp + Za\,(Zb + Zc)/(Za + Zb + Zc) \quad (2\text{-}16b)$$
$$Z12 = Za\,(Zb + Zc)/(Za + Zb + Zc) \quad (2\text{-}16c)$$
$$Z13 = Za\,Zb/(Za + Zb + Zc) \quad (2\text{-}16d)$$
$$Z33 = Zq + Zb\,(Zc + Za)/(Za + Zb + Zc) \quad (2\text{-}16e)$$
$$Z34 = Zb\,(Zc + Za)/(Za + Zb + Zc) \quad (2\text{-}16f)$$

From Eqs. (2-16), each element in the equivalent circuit is obtained :

$$Za = (Z13^2 - Z12\,Z34)/(Z13 - Z34) \quad (2\text{-}17a)$$
$$Zb = (Z13^2 - Z12\,Z34)/(Z13 - Z12) \quad (2\text{-}17b)$$
$$Zc = - (Z13^2 - Z12\,Z34)/Z13 \quad (2\text{-}17c)$$
$$Zp = Z11 - Z12 \quad (2\text{-}17d)$$
$$Zq = Z33 - Z34 \quad (2\text{-}17e)$$

The derivation of Eqs. (2-16) and (2-17) are detailed in Appendix C. As computation for the Z parameters, accordingly the parameters in Eqs. (2-17), it is convenient to use normalized impedance parameters with respect to the characteristic impedance of each stripline. Thus, the matrix in Eq. (2-5) may be modified as follows :

$$\begin{bmatrix} v1 \\ v2 \\ v3 \\ v4 \end{bmatrix} = \begin{bmatrix} z11 & z12 & z13 & z13 \\ z12 & z11 & z13 & z13 \\ z13 & z13 & z33 & z34 \\ z13 & z13 & z34 & z33 \end{bmatrix} \begin{bmatrix} i1 \\ i2 \\ i3 \\ i4 \end{bmatrix} \quad (2\text{-}18a)$$

where

[v] : normalized voltage vector

[i] : normalized current vector

$$[\,v\,] = \mathrm{diag}\,[\,\sqrt{Zo\mathit{i}}\,]^{-1}[\,V\,] \quad (2\text{-}18b)$$
$$[\,i\,] = \mathrm{diag}\,[\,\sqrt{Zo\mathit{i}}\,]\;\;[\,I\,] \quad (2\text{-}18c)$$
$$zmn = \frac{Zmn}{\sqrt{Zom}\,\sqrt{Zon}} \quad (2\text{-}18d)$$

m = 1, 2, 3, 4
n = 1, 2, 3, 4

Zoi : Stripline characteristic impedance at the i-th port

The normalized terminal impedance *zi*'s are given by Eqs. (2-15). Small letters z, v, i for Z, V, I denote normalized quantities. Accordingly, the equations in Eqs. (2-8), (2-9) and (2-13) are replaced by normalized ones, - replacing Z's by z's - , and solved with

respect to normalized parameters. Finally, equivalent circuit parameters in Eq. (2-17) are solved using the relation of Eq. (2-18d).

The S matrix for a 4-port network is obtained form the relation

$$[\,S\,] = [\,[\,z\,] + [\,U\,]\,]^{-1}[\,[\,z\,] - [\,U\,]\,] \qquad (2\text{-}19)$$

where [z] is a normalized impedance matrix and [U] is a unit matrix. The matrix relations are detailed in Appendix D.

3. FIELD ANALYSIS

3-1. Modes in a Rectangular Waveguide

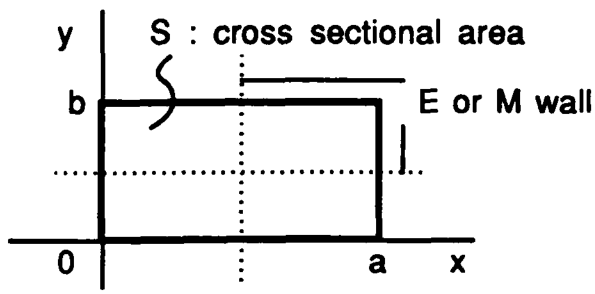


Fig.3-1 Rectangular waveguide

Fig. 3-1 shows a rectangular waveguide structure with the reference coordinate. With the scalar potentials, the general field expressions for TE to z and TM to z modes are as follows [4] :

TE	TM	
$\mathbf{E}\,t = -\nabla t\,\Psi \times \mathbf{z}$	$\mathbf{H}\,t = \nabla t\,\varphi \times \mathbf{z}$	
$\mathbf{H}\,t = -\dfrac{j\beta}{z\wedge}\,\nabla t\,\psi$	$\mathbf{E}\,t = -\dfrac{j\beta}{y\wedge}\,\nabla t\,\varphi$	(3-1)
$H\,z = \dfrac{kc^2}{z\wedge}\,\psi$	$E\,z = \dfrac{kc^2}{y\wedge}\,\varphi$	

where

$$kc^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

m, n : see Eqs. (3-2)

$$\beta^2 = k^2 - kc^2$$

$$y\wedge = j\,\omega\,\varepsilon, \quad z\wedge = j\,\omega\,\mu, \quad k^2 = -\,y\wedge\,z\wedge = \omega^2\,\varepsilon\,\mu$$

ε : permitivity μ : permeability

$$\mathbf{E}\,t = \mathbf{x}\,Ex + \mathbf{y}\,Ey$$

$$\mathbf{H}\,t = \mathbf{x}\,Hx + \mathbf{y}\,Hy$$

$$\nabla t = \mathbf{x}\frac{\partial}{\partial x} + \mathbf{y}\frac{\partial}{\partial y}$$

x, y, z : unit vector in x, y, z direction

where z dependance $e^{-j\beta z}$ is assumed and subscript t denotes the transverse components of a quantity. Assuming an electric wall or a magnetic wall at each center of the wave guide, one can express scalar potentials including conductive guide boundary conditions as follows :

$$\Psi_{mn} = P_{mn} \cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right) \qquad m,n = 0,1,2,3,\text{--} \qquad (3\text{-}2a)$$

$$\varphi_{mn} = P_{mn} \sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \qquad m,n = 1,2,3,\ \text{--} \qquad (3\text{-}2b)$$

$$P_{mn} = \sqrt{\frac{\delta_m \delta_n}{a\,b}}\ \frac{1}{k_c} \qquad \delta_i = \begin{cases} 1 & i = 0 \\ 2 & i \neq 0 \end{cases}$$

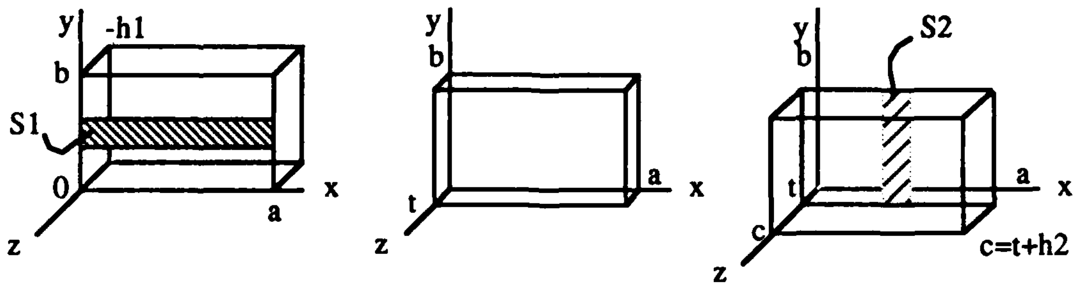
where subscripts mn corresponds to the order of a harmonics, and Pmn is a coefficient to normalize the norm of scalar potential vector functions so as

$$\int_S |\nabla_t \Psi_{mn}|^2\, dxdy = 1 \qquad (3\text{-}3a)$$

$$\int_S |\nabla_t \Psi_{mn}|^2\, dxdy = 1\,. \qquad (3\text{-}3b)$$

Even symmetry solutions with respect to the x or y axis are obtained for m or n being an odd integer. Similarly, odd solutions are obtained for m or n being an even integer.

3-2. Field Expansions inside the Cavity



(a) Region (1) $-h1 \le z \le 0$ (b) Region (3) $0 \le z \le t$ (c) Region (2) $t \le z \le c$

Fig. 3-2 Subregions for field analysis

The resonant structure of Fig. 1-1 can be subdivided into three homogeneous regions as shown in Fig. 3-2. In each region, we can expand the electromagnetic field in terms of TE-to-z and TM-to-z waveguide modes. Incorporating the boundary conditions for z=-h1 and z=c, the following expressions can be assumed :

region (1) : $-h1 \le z \le 0$

$$Et^{(1)} = -\sum_{mn}^{MN} [\, Amn^{(1)} \nabla t\, \Psi mn \times z + Bmn^{(1)} (\beta mn^{(1)} / y\hat{}_1)\, \nabla t\, \varphi mn \,] \sin \beta mn^{(1)} (z+h1) \qquad (3\text{-}4a)$$

$$Ht^{(1)} = \sum_{mn}^{MN} [\, Amn^{(1)} (\beta mn^{(1)} / z\hat{}_1)\, \nabla t\, \Psi mn + Bmn^{(1)}\, \nabla t\, \varphi mn \times z] \cos \beta mn^{(1)} (z+h1) \qquad (3\text{-}4b)$$

$$\beta mn^{(1)} = (\, k_1{}^2 - kc^2 \,)^{1/2} \qquad (3\text{-}4c)$$

region (2) : $t \le z \le c$

$$Et^{(2)} = -\sum_{mn}^{MN} [\, Amn^{(2)} \nabla t\, \Psi mn \times z + Bmn^{(2)} (\beta mn^{(2)} / y\hat{}_2)\, \nabla t\, \varphi mn \,] \sin \beta mn^{(2)} (z-c) \qquad (3\text{-}5a)$$

$$Ht^{(2)} = \sum_{mn}^{MN} [\, Amn^{(2)} (\beta mn^{(2)} / z\hat{}_2)\, \nabla t\, \Psi mn + Bmn^{(2)}\, \nabla t\, \varphi mn \times z \} \cos \beta mn^{(2)} (z-c) \qquad (3\text{-}5b)$$

$$\beta mn^{(2)} = (\, k_2{}^2 - kc^2 \,)^{1/2} \qquad (3\text{-}5c)$$

region (3) : $0 \le z \le t$

$$Et^{(3)} = -\sum_{mn}^{MN} [\, \{ Cmn \sin \beta mn^{(3)} z + Dmn \sin \beta mn^{(3)} (z-t) \}\, \nabla t\, \Psi mn \times z + \{ Fmn \sin \beta mn^{(3)} z + Gmn \sin \beta mn^{(3)} (z-t) \}\, (\beta mn^{(2)} / y\hat{}_3)\, \nabla t\, \varphi mn \qquad (3\text{-}6a)$$

$$Ht^{(3)} = \sum_{mn}^{MN} [\, \{ Cmn \cos \beta mn^{(3)} z + Dmn \cos \beta mn^{(3)} (z-t) \}\, (\beta mn^{(3)} / z\hat{}_3)\, \nabla t\, \Psi mn + \{ Fmn \cos \beta mn^{(3)} z + Gmn \cos \beta mn^{(3)} (z-t) \}\, \nabla t\, \varphi mn \times z \,] \qquad (3\text{-}6b)$$

$$\beta mn^{(3)} = (\, k_3{}^2 - kc^2 \,)^{1/2} \qquad (3\text{-}6c)$$

where $Amn^{(1)}$, $Amn^{(2)}$, $Bmn^{(1)}$, $Bmn^{(2)}$, Cmn, Dmn, Fmn, and Gmn are unknown coefficients, and superscripts in parentheses refer to respective regions. The boundary conditions at each interface are

Interface (I) : at $z = 0$

$$Et_1 = Et_3 \qquad \text{on } S \qquad (3\text{-}7a)$$

$$Et_1 = Et_3 = 0 \qquad \text{on } S1 \qquad (3\text{-}7b)$$

$$Ht_3 - Ht_1 = \begin{cases} \Delta Ht^{(1)} & \text{on } S1 \\ 0 & \text{on } S\text{-}S1 \end{cases} \qquad (3\text{-}7c)$$

Interface (II) : at $z = t$

$$Et_2 = Et_3 \qquad \text{on } S \qquad (3\text{-}8a)$$

$$Et_2 = Et_3 = 0 \qquad \text{on } S2 \qquad (3\text{-}8b)$$

$$Ht_2 - Ht_3 = \begin{cases} \Delta Ht^{(2)} & \text{on } S2 \\ 0 & \text{on } S\text{-}S2 \end{cases} \qquad (3\text{-}8c)$$

where $\Delta Ht^{(i)}$ ($i = 1,2$) are unknown functions of x, y. These functions are expanded in terms of a set of known orthogonal vector functions $\chi v^{(i)}$ defined over Si with unkown coefficients $Pv^{(i)}$ (see next section) :

$$\Delta Ht^{(1)} = \sum_{v} Pv^{(1)} \chi v^{(1)} \qquad (3\text{-}9a)$$

$$\Delta Ht^{(2)} = \sum_{v} Pv^{(2)} \chi v^{(2)} \qquad (3\text{-}9b)$$

Applying the boundary conditions of Eqs. (3-7), (3-8) to Eqs. (3-4), (3-5), (3-6) and making use of the orthogonal properties of $\nabla t\Psi mn$ and $\nabla t\varphi mn$, we obtain a homogeneous system of equations in terms of unknown coefficient Pv :

$$\sum_{v_1}^{v_1 M} Pv^{(1)}\, U\mu v^{(11)} + \sum_{v_2}^{v_2 M} Pv^{(2)}\, U\mu v^{(12)} = 0 \qquad \mu = 1, 2, 3, \; v_1 M \qquad (3\text{-}10a)$$

$$\sum_{v_1}^{v_1 M} Pv^{(1)}\, U\mu v^{(21)} + \sum_{v_2}^{v_2 M} Pv^{(2)}\, U\mu v^{(22)} = 0 \qquad \mu = 1, 2, 3, \; v_2 M \qquad (3\text{-}10b)$$

where

$$U\mu v^{(ij)} = \sum_{mn}^{MN} S_i \{ amn^{(ij)}\, \xi mn,\mu^{(i)}\, \xi mn,v^{(j)} - (\beta^{(i)}/y\hat{}_i)\, bmn^{(ij)}\, \theta mn,\mu^{(i)}\, \theta mn,v^{(j)} \} \qquad (3\text{-}10c)$$

$$i, j = 1, 2$$

$$\xi_{mn,\mu^{(i)}} = \int_{Si} \chi_{\mu^{(i)}} \bullet \nabla_t \Psi_{mn}\, ds \qquad i = 1, 2 \qquad (3\text{-}10d)$$

$$\theta_{mn,\mu^{(i)}} = \int_{Si} \chi_{\mu^{(i)}} \bullet \nabla_t \varphi_{mn} \times \mathbf{z}\, ds \qquad i = 1, 2 \qquad (3\text{-}10e)$$

$$amn^{(11)} = Ka\,(\,1/S_1\,)\,(\,\beta^{(2)}Ct_2 + \beta^{(3)}Ct_3\,) \qquad (3\text{-}10f)$$

$$amn^{(12)} = Ka\,(\,1/\,(S_1S_3)\,)\,\beta^{(3)} \qquad (3\text{-}10g)$$

$$amn^{(21)} = -\,Ka\,(\,1/\,(S_2S_3)\,)\,\beta^{(3)} \qquad (3\text{-}10h)$$

$$amn^{(22)} = -\,Ka\,(\,1/S_2\,)\,(\,\beta^{(1)}Ct_1 + \beta^{(3)}Ct_3\,) \qquad (3\text{-}10i)$$

$$Ka = z^\wedge / \{\beta^{(3)2} - \beta^{(3)}Ct_3(\beta^{(2)}Ct_2 + \beta^{(1)}Ct_1) - \beta^{(1)}\beta^{(2)}Ct_1Ct_2\,\}$$

$$bmn^{(11)} = Kb\,(\,1/S_1\,)\,(\beta^{(3)}Ct_2 + (\varepsilon_3/\varepsilon_2)\beta^{(2)}Ct_3) \qquad (3\text{-}10j)$$

$$bmn^{(12)} = Kb\,(\,1/\,(S_1S_3)\,)\,(\varepsilon_3/\varepsilon_2)\,\beta^{(2)} \qquad (3\text{-}10k)$$

$$bmn^{(21)} = -\,Kb\,(\,1/\,(S_2S_3)\,)\,(\varepsilon_3/\varepsilon_1)\,\beta^{(1)} \qquad (3\text{-}10l)$$

$$bmn^{(22)} = -\,Kb\,(\,1/S_2\,)\,(\beta^{(3)}Ct_1 + (\varepsilon_3/\varepsilon_1)\beta^{(1)}Ct_3) \qquad (3\text{-}10m)$$

$$Kb = \beta^{(3)} / [\,(\varepsilon_3/\varepsilon_1)(\varepsilon_3/\varepsilon_2)\beta^{(1)}\beta^{(2)} - \beta^{(3)}Ct_3\{(\varepsilon_3/\varepsilon_1)\beta^{(1)}Ct_2 + (\varepsilon_3/\varepsilon_2)\beta^{(2)}Ct_1\}\, - \beta^{(3)2}Ct_1Ct_2\,]$$

$$\beta^{(i)} = \beta mn^{(i)} \qquad i = 1, 2, 3$$

$$C_1, C_2, C_3 = \cos\beta^{(1)}h1,\ \cos\beta^{(2)}h2,\ \cos\beta^{(3)}t$$

$$S_1, S_2, S_3 = \sin\beta^{(1)}h1,\ \sin\beta^{(2)}h2,\ \sin\beta^{(3)}t$$

$$Ct_1, Ct_2, Ct_3 = C_1/S_1,\ C_2/S_2,\ C_3/S_3$$

$$z^\wedge = j\,\omega\,\mu_o \qquad \mu_o : \text{free space permeability} = 4\pi\times10^{-7}[\text{H/m}]$$

The derivations of Eqs. (3-10) are detailed in Appendix E. The condition for non-trivial solution determines the characteristic equation of the given structure. This equation may be regarded as a function of ω, $l1$, $l3$ equated to zero :

$$f\,(\,\omega, l1, l3\,) = 0 \qquad (3\text{-}11)$$

For given value of $\omega = \omega_r$, Eq. (3-11) can be solved to evaluate the different pairs of $l1$ and $l3$ giving rise to the same resonant frequency ω_r. These values of $l1$ and $l3$ can be used for computing the discontinuity parameters discussed in the previous section.

3-3. H Field (Current) Expansion on the Strips

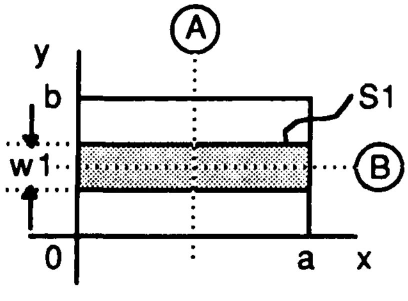
The H field on the strip are expanded in terms of known basis functons χ_v with unknown coefficients Pv. Actually what will be expanded is the H field discontinuity which is equal to $\mathbf{Jt} \times \mathbf{z}$, viz.

$$\Delta \mathrm{Ht} = \sum_{v}^{vmax} Pv\, \chi_v = \mathbf{Jt} \times \mathbf{z} \tag{3-12}$$

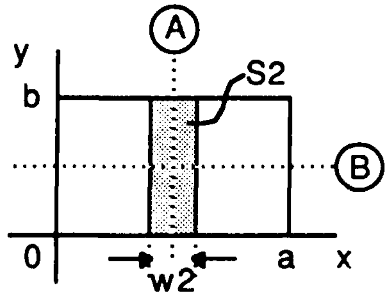
The basis functions are chosen in such a way that the field is non-zero only on the strip. Additional boundary condition must be satisfied at the strip ends where the striplines are terminated with electric walls. The field may be expanded in terms of the basis functions of two different types. One of them is harmonic basis functions and another is singular basis functions. The singular behavior of the magnetic field component normal to the stripline edges is incorporated in the singular basis functions. They are therefore expected to provide a faster numerical convergence. For the H field expansion on S1 [see Fig. 3-3(a)], the following set is employed :

$$\chi_{x,rs}^{(1)} = \sin \frac{r\pi x}{a} \sin \left[\frac{s\pi}{w1} \left\{y - \frac{(b-w1)}{2}\right\}\right] \tag{3-13a}$$

$$\chi_{y,rs}^{(1)} = \cos \frac{r\pi x}{a} \frac{\cos \left[\frac{s\pi}{w1} \left\{ y - \frac{(b-w1)}{2} \right\}\right]}{\sqrt{1 - \left\{ \frac{(y-b/2)}{w1/2} \right\}^2}} \tag{3-13b}$$



(a) at interface (I)



(b) at interface (II)

Fig. 3-3 Striplines at each interface

where subscript rs is used instead of v for representing the variations in the two orthogonal directions (see Appendix F). Similarly, for the H field on S2 in Fig. 3-3(b), the expansions are

$$\chi_{x,rs}^{(2)} = \frac{\cos\left[\frac{r\pi}{w2}\left\{x - \frac{(a-w2)}{2}\right\}\right]}{\sqrt{1 - \left\{\frac{(x-a/2)}{w2/2}\right\}^2}} \cos\frac{s\pi y}{b} \qquad (3\text{-}14a)$$

$$\chi_{y,rs}^{(2)} = \sin\left[\frac{r\pi}{w2}\left\{x - \frac{(a-w2)}{2}\right\}\right] \sin\frac{s\pi y}{b} \qquad (3\text{-}14b)$$

Depending on whether an electric or magnetic wall is assumed to be at (A) and (B) in Fig. 3-3(a), the numbers r and s in Eqs. (3-13) are classified to be odd or even as illustrated in Table 3-1.

Table 3-1 rs-table with respect to E M walls

(a) at interface (I)

	r	s
M wall	2r′+1	2s′
E wall	2r′	2s′ +1

$$\chi_x^{(1)}: r, s = \begin{cases} 1, 3, 5, \text{--} \\ \text{or} \\ 2, 4, 6, \text{--} \end{cases}$$

$$\chi_y^{(1)}: r, s = \begin{cases} 1, 3, 5, \text{--} \\ \text{or} \\ 0, 2, 4, \text{--} \end{cases}$$

(b) at interface (II)

	r	s
M wall	2r′	2s′+1
E wall	2r′+1	2s′

$$\chi_x^{(2)}: r, s = \begin{cases} 1, 3, 5, \text{--} \\ \text{or} \\ 0, 2, 4, \text{--} \end{cases}$$

$$\chi_y^{(2)}: r, s = \begin{cases} 1, 3, 5, \text{--} \\ \text{or} \\ 2, 4, 6, \text{--} \end{cases}$$

4. COMPUTED RESULTS

In accordance with the technique described above, the electromagnetic fields in each region are expressed in termes of the series expansions. In the numerical computation, only a finite number of terms can be retained in the series expansions. In order to obtain a proper convergent behavior of the solution, the number of the terms was chosen in such a way that the highest spatial frequencies of the electromagnetic field are approximately the same in the transverse directions (x,y). The current on a stripline is expanded in such a way that the distributions of each component (Jx, Jy) in the transverse direction are expressed by only the first term and that the distributions in the longitudinal direction by a set of terms, the number of which are the same on each stripline.

The method is first tested by computing the resonant frequency of the isolated stripline with certain structural parameters. Fig. 4-1 shows the convergence of the resonant frequency with respect to the number of the terms of the field expasions in the homogeneous region of Fig. 3-2. The number MN was chosen to be the same in both x and y directions. The results exhibit very good agreement with those by the SDA [5] in which the resonant frequency was calculated from the equation

$$fo = \frac{c}{a}\,\frac{\beta o}{\beta} \qquad (4\text{-}1)$$

where c : speed of the light

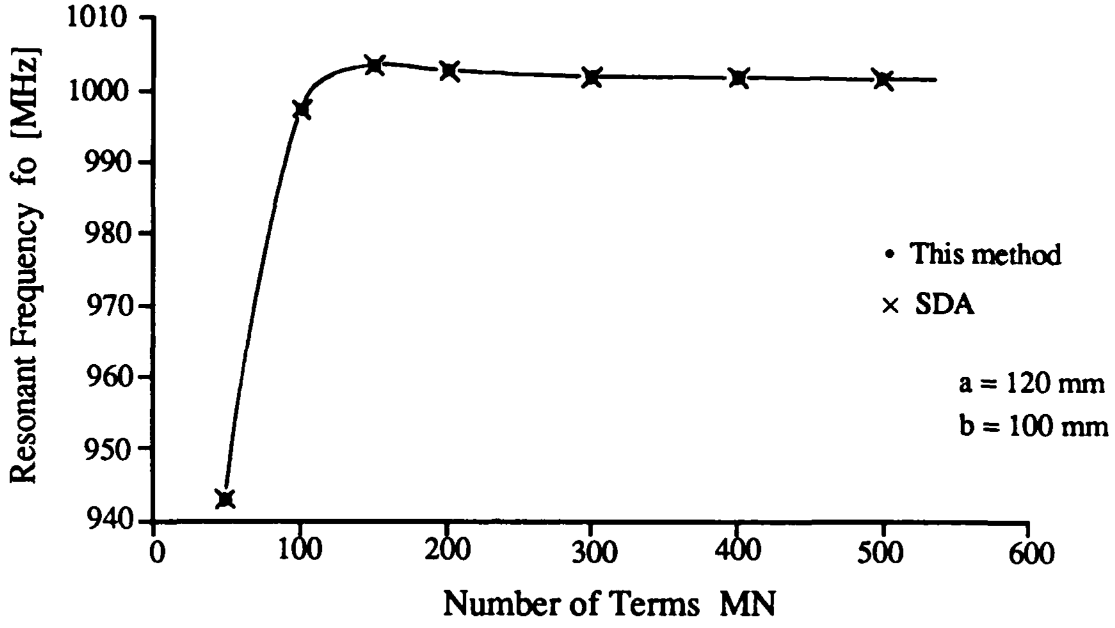
a : stripline length

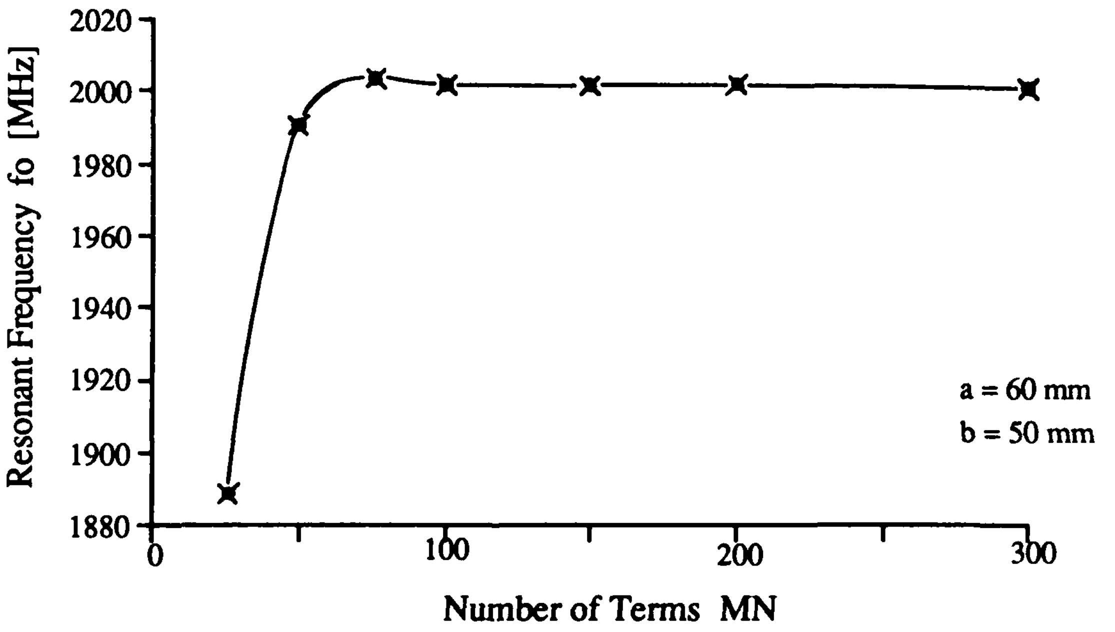
ß : propagtion constant in proximity of fo

ßo : 〃 in free space

because computationaly the field was represented by the same number of harmonics in both this method and SDA.

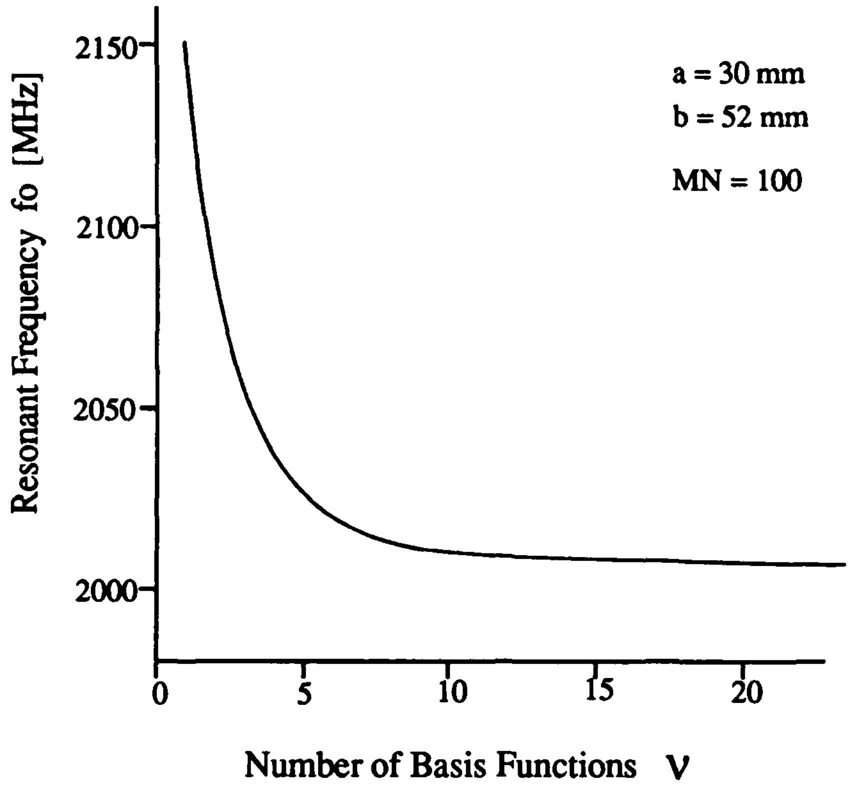
Fig. 4-2 shows the convergence of the resonant frequency of crossing stripline structure with respect to the number v of the current expansion terms in the longitudinal direction. At beginning, we expected that the number of the terms would be required as





h1 = h2 = 5 mm, t = 1mm, w1 = w2 = 1 mm, $\varepsilon r = 3.8$

Fig. 4-1 Convergence of the resonant frequency of an isolated stripline with respect to the number of terms for electromagnetic field in the x and y directions



h1 = h2 = 5 mm, t = 1mm

w1 = w2 = 1 mm, εr = 3.8

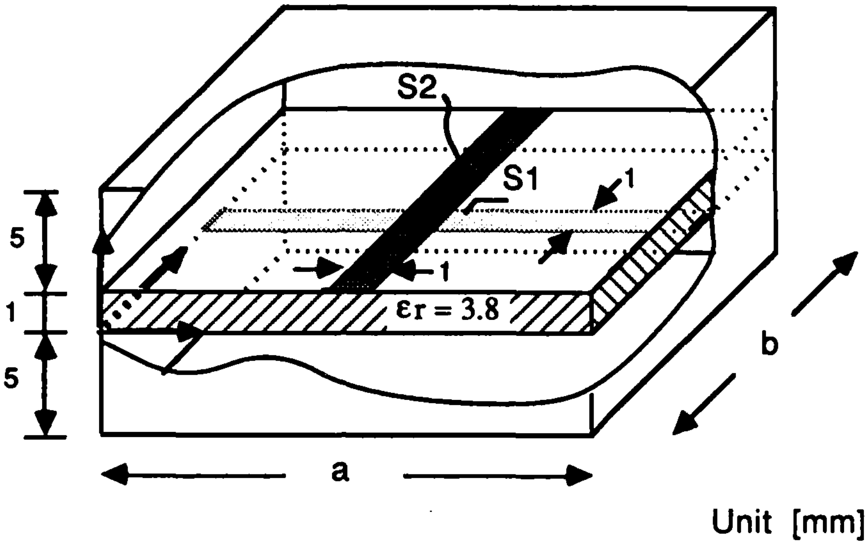
Fig. 4-2 Convergence of the resonant frequency of crossing striplines with respect to the number of basis functions

large as that of the electromagnetic field so that the longitudinal current would have the same highest spatial frequency and the results would have a good convergence. From Fig. 4-2, however, we could observe a fast convergence characteristic with a relatively small number of current expansions if the electromagnetic field is well expressed by an adequate number of harmonics.

Fig. 4-3 shows the parameters of the structure to be computed. The substrate was placed symmetrically between the top and bottom planes with both strips having the same widths. Hence, the impedance matrix representation of the discontinuities has z11=z33 and z12=z34, i.e. unknown parameters number is three: z11,z12, z13. The computation was done at the three different frequencies: 0.5, 1, 2GHz. The approximate lengths of striplines, namely the size of cavity at each frequency, are also illustrated in Fig. 4-3. The reference planes are defined in the same mannar as illustrated in Fig. 2-8.

Table 4-1 shows the element values for the equivalent circuit of the discontinuities shown in Fig. 4-4. Fig. 4-5 shows the corresponding S parameters of the discontinuities. Note that the elements for Za and Zb (=Za) are capacitors with negative value. This is acceptable because they compensate the parallel distributed capacitance for the isolated stripline with the absence of the other stripline as follows. Assuming the absence of stripline 2 by making the port 3 and 4 open, the structure is nearly an isolated stripline. The section between port1 and 2, therefore, would have an equivalent parallel capacitor which is coresponding to the distributed capacitor of the stripline. When the section is very short, the capacitance of the equivalent capacitor may be negative so that it would compensate the fringing capacitance due to the open ends at port 3 and 4 whose value could be larger than that of the distributed capacitor as a simple stripline section.

Zc was also calculated with h1=10mm, h2=100mm, t=5mm, w1=w2=0.4mm, ε_r=1at 2 GHz simulating the crossing model of the work by Giri, et al [6]. The result for the value of Cc was 0.082pF, while the value of the coupling capacitor Cm was estimated to be 0.15pF by using the equivalent radius.



a ≈ 230 ~ 240
b ≈ 200 ~ 210 @ 0.5 GHz

a ≈ 100 ~ 120
b ≈ 90 ~ 100 @ 1 GHz

a ≈ 30 ~ 40
b ≈ 50 ~ 52 @ 2 GHz

Fig. 4-3 Structural parameters for computation

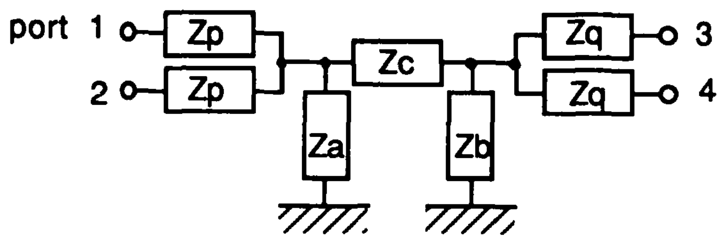


Fig. 4-4 Equivalent circuit of the discontinuities

Table 4-1 Values of elements

	0.5 GHz	1 GHz	2 GHz
Lp=Lq	0.331nH	0.331nH	0.329nH
Ca=Cb	-0.0885pF	-0.103pF	-0.101pF
Cc	0.249pF	0.272pF	0.258pF
MN V	400 20	200 20	100 20

$$Lp = Lq = \frac{Zo\,Zp}{j\,\omega}$$

$$Ca = Cb = \frac{1}{j\,\omega\,Zo\,Za}$$

$$Cc = \frac{1}{j\,\omega\,Zo\,Zc}$$

Zp, Zq, Za, Zb, Zc : Normalized impedance

Zo : Stripline characteristic impedance
(159.2, 159.1, 158.3Ω at 0.5, 1, 2 GHz)

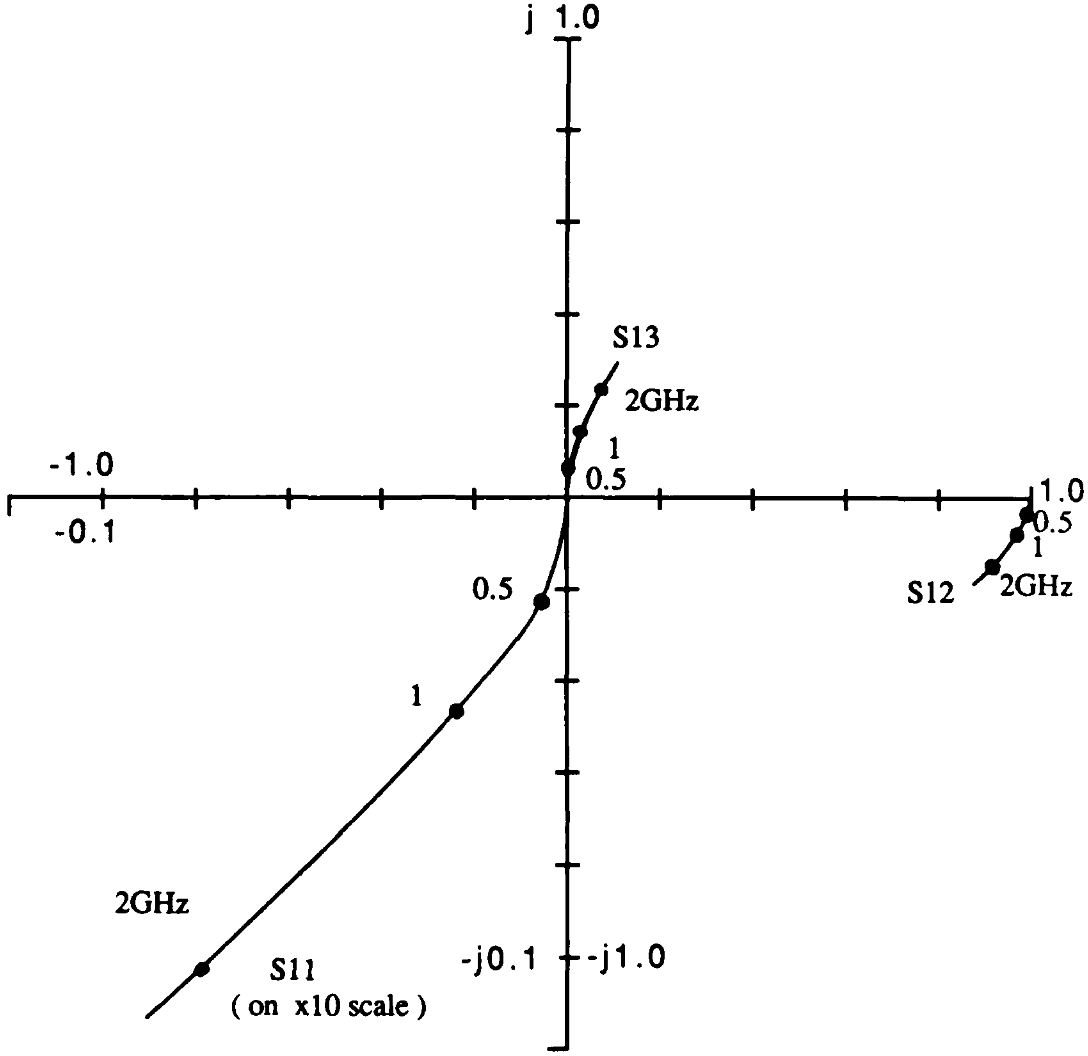


Fig. 4-5 S parameters of the discontinuities of the structure

Longitudinal Currents (Jy on Stripline 1)

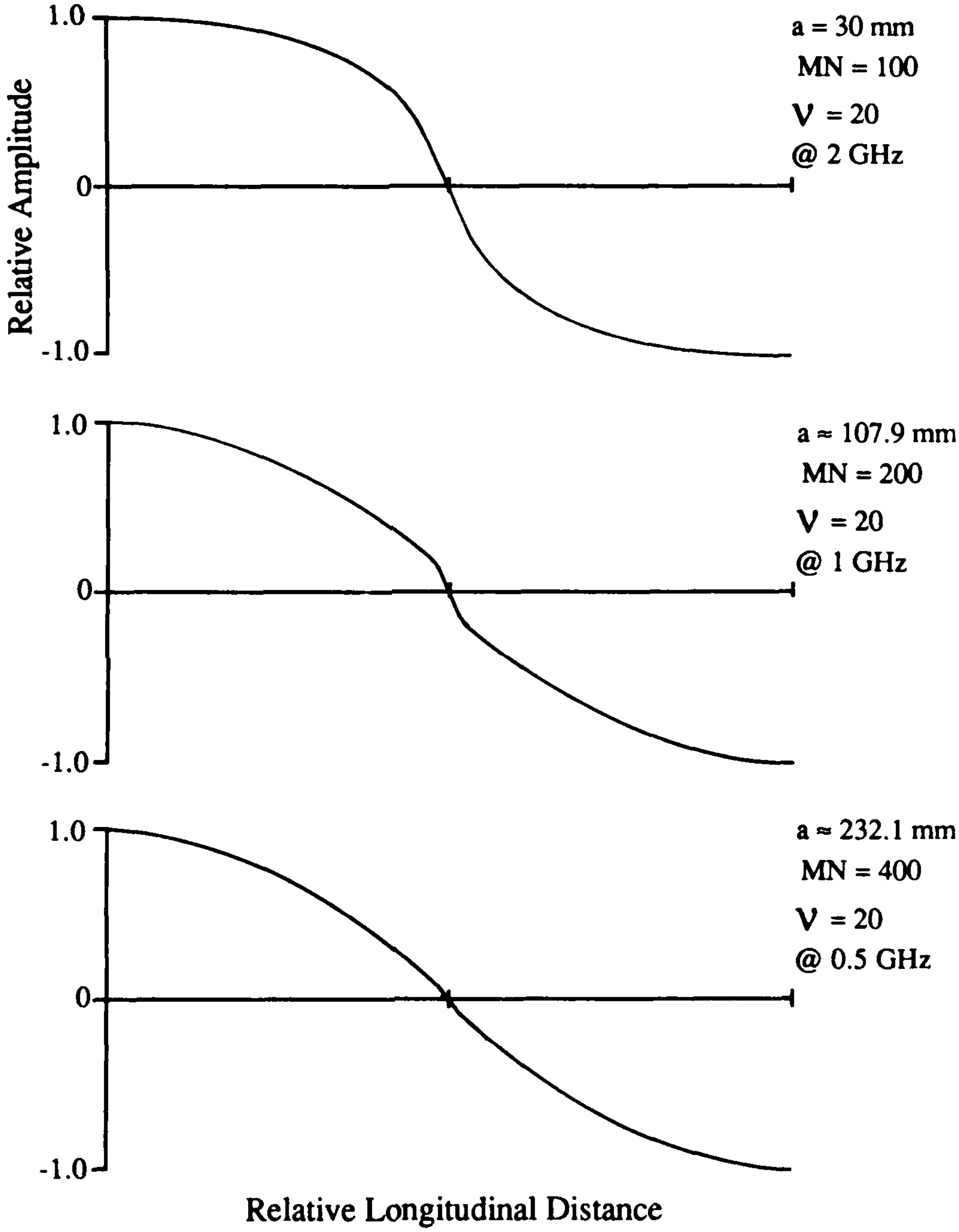


Fig. 4-6 Current distribution on the stripline

(a) longitudinal current

Transverse Currents (Jx on Stripline 1)

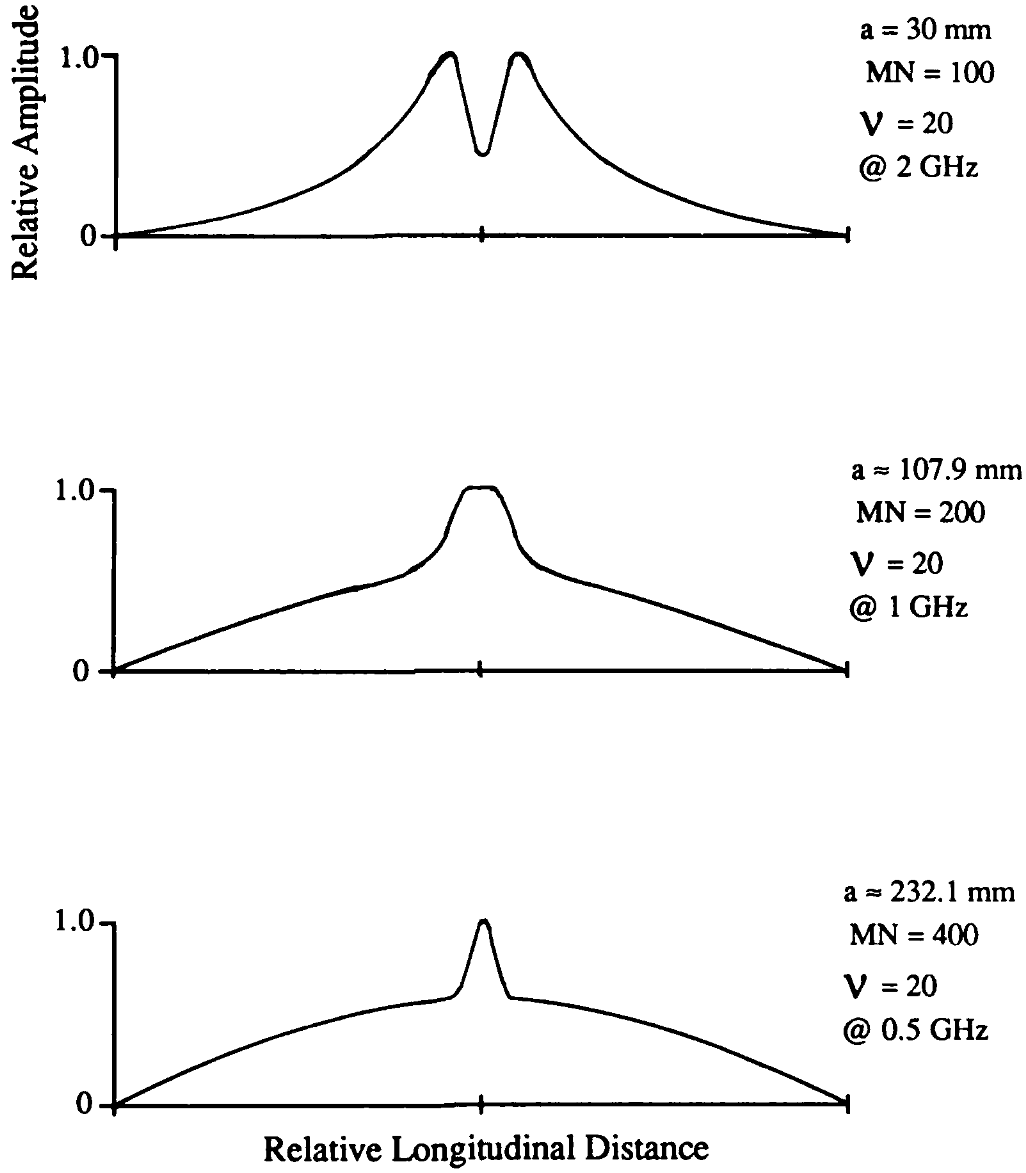
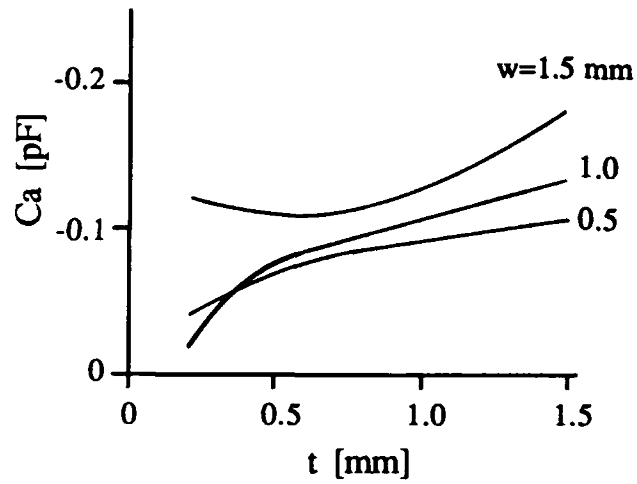
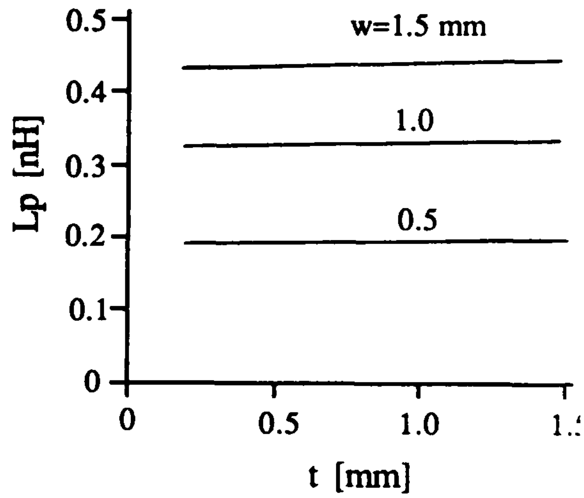


Fig. 4-6 Current distribution on the stripline

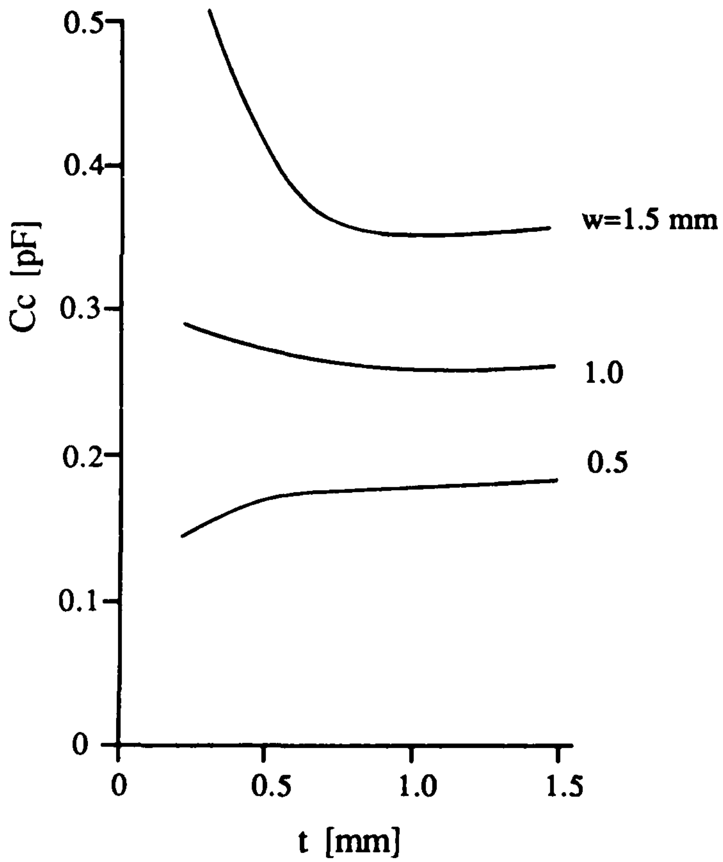
(b) transverse current



(a) Ca



(b) Lp



(c) Cc

Fig. 4-7 Element values vs stripline width and substrate thickness

Fig. 4-6(a),(b) show the longitudinal and transverse components of the current densities at the center of the strip at the three different frequencies. In Fig. 4-6(a), it is observed that each figure was of perturbated cosine form and had larger absolute amplitude than cosine in the proximity of the center. This feature is anticipated from the insight that the stripline whose length is slightly shorter than a half wavelength turn out to be a resonator with capacitive loading at the center of the stripline. The longitudinal current, hence, may be represented by the combination of a cosine and an additional polynomial functions [7] so that the computation time could be reduced. As to the transverse current shown in Fig. 4-6(b), the figures were not readily characterized. This implies that the current function would require a number of terms of basis functions; the current function could not be easily replaced by few terms of special functions.

Fig. 4-7(a)(b)(c) show the computed results for different values of the stripline width and the substrate thickness.

5. CONCLUSIONS

A method of analysis has been described for characterizing the discontinuities of two crossed striplines. The method is based on a generalized transverse resonance technique for computing the resonant frequency of a resonator created by enclosing the crossing with auxiliary perfectly conducting walls. This resonator problem is analyzed as the waveguide scattering for waves traveling in the direction normal to the substrate surface. For a specified frequency, resonant structures are found by adjusting the lengths of the strips and hence the resonator size. These structures are used for deriving the equivalent circuit parameters characterizing the discontinuity.

This method can also be applied for the characterization of stripline-slotline transition.

APPENDIX

A. DERIVATION OF EQ. (2-6)

det || [Z] + diag [Zi] ||

$$= \begin{bmatrix} Z11+Z1 & Z12 & Z13 & Z13 \\ Z12 & Z11+Z2 & Z13 & Z13 \\ Z13 & Z13 & Z33+Z3 & Z34 \\ Z13 & Z13 & Z34 & Z33+Z4 \end{bmatrix} \quad \begin{array}{l} \text{Subtract row 2 from row 1} \\ \text{Subtract row 3 from row 4} \end{array}$$

$$= \begin{bmatrix} Z11+Z1-Z12 & Z12-Z11-Z2 & 0 & 0 \\ Z12 & Z11+Z2 & Z13 & Z13 \\ Z13 & Z13 & Z33+Z3 & Z34 \\ 0 & 0 & Z34-Z33-Z3 & Z33+Z4-Z34 \end{bmatrix} \quad \begin{array}{l} \text{Subtract column 2} \\ \text{from column 1} \\ \text{Subtract column 3} \\ \text{from column 4} \end{array}$$

$$= \begin{bmatrix} 2Z11-2Z12+Z1+Z2 & Z12-Z11-Z2 & 0 & 0 \\ Z12-Z11-Z2 & Z11+Z2 & Z13 & 0 \\ 0 & Z13 & Z33+Z3 & Z34-Z33-Z3 \\ 0 & 0 & Z34-Z33-Z3 & 2Z33-2Z34+Z3+Z4 \end{bmatrix}$$

$$\begin{aligned} = \; & \{ (2Z11-2Z12+Z1+Z2)(Z11+Z2) - (Z12-Z11-Z2)^2 \} \\ & \times \{ (2Z33-2Z34+Z3+Z4)(Z33+Z3) - (Z34-Z33-Z3)^2 \} \\ & - 4\, Z13^2 \{ Z11-Z12+(Z1+Z2)/2 \} \{ Z33-Z34+(Z3+Z4)/2 \} \end{aligned}$$

$$\begin{aligned} = \; & \{ (Z11+Z1)(Z11+Z2)-Z12^2 \} \{ (Z33+Z3)(Z33+Z4)-Z34^2 \} \\ & - 4\, Z13^2 \{ Z11-Z12+(Z1+Z2)/2 \} \{ Z33-Z34+(Z3+Z4)/2 \} \end{aligned}$$

where the following matrix relation is used :

$$\begin{bmatrix} a11 & a12 & 0 & 0 \\ a21 & a22 & c & 0 \\ 0 & c & b11 & b12 \\ 0 & 0 & b21 & b22 \end{bmatrix} = [A][B] - c^2\, a11\, b22$$

where

$$[A] = \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix} \qquad [B] = \begin{bmatrix} b11 & b12 \\ b21 & b22 \end{bmatrix}$$

B. EIGENVECTOR FOR AN ODD OR EVEN RESONANCE

Insert the odd resonance condition

$$Z11 - Z12 + Z1 = 0 \tag{B-1}$$

into Eq. (2-3) to obtain

$$Z12\,(I1 + I2) + Z13\,(I3 + I4) = 0 \tag{B-2a}$$

$$Z12\,(I1 + I2) + Z13\,(I3 + I4) = 0 \tag{B-2b}$$

$$Z13\,(I1 + I2) + (Z33 + Z3)\,I3 + Z34\,I4 = 0 \tag{B-2c}$$

$$Z13\,(I1 + I2) + Z34\,I3 + (Z33 + Z3)\,I4 = 0\ . \tag{B-2d}$$

Subtracting Eq. (B-2d) from Eq. (B-2c), we obtain

$$(Z33 + Z3 - Z34)\,I3 - (Z33 + Z3 - Z34)\,I4 = 0$$

or

$$I3 = I4\ . \tag{B-3}$$

Substitution of Eq. (B-3) into Eqs. (B-2) yields

$$\begin{bmatrix} Z12 & 2Z13 \\ 2Z13 & Z33+Z34+Z3 \end{bmatrix} \begin{bmatrix} I1+I2 \\ I3 \end{bmatrix} = 0\ . \tag{B-4}$$

As Eq. (B-4) is a homogeneous system and is valid for the arbitrary value of Z3, the current vector is identical to zero, i.e.,

$$I1 + I2 = 0$$

$$I3 = 0$$

hence,

$$I1 = -I2 \tag{B-5a}$$

$$I3 = (I4) = 0\ . \tag{B-5b}$$

Similarly, the eigenvector for another odd resonance is obtained as

$$I3 = -I4 \tag{B-6a}$$

$$I1 = (I2) = 0\ . \tag{B-6b}$$

The eigenvector for the even resonance may be obtained from Eq. (2-3) as Z1 = Z2 and Z3 = Z4 :

$$(Z11 + Z1)\, I1 + Z12\, I2 + Z13\, (I3 + I4) = 0 \quad \text{(B-7a)}$$

$$Z12\, I1 + (Z11 + Z1)\, I2 + Z13\, (I3 + I4) = 0 \quad \text{(B-7b)}$$

$$Z13\, (I1 + I2) + (Z33 + Z3)\, I3 + Z34\, I4 = 0 \quad \text{(B-7c)}$$

$$Z13\, (I1 + I2) + Z34\, I3 + (Z33 + Z3)\, I4 = 0 \quad \text{(B-7d)}$$

Subtracting Eq. (B-7b) from Eq. (B-7a) and so Eq. (B-7d) from Eq. (B-7c), we get

$$(Z11 + Z1 - Z12)\, (I1 - I2) = 0 \quad \text{(B-8a)}$$

$$(Z33 + Z3 - Z34)\, (I3 - I4) = 0\ . \quad \text{(B-8b)}$$

Since the even resonance condition does not include the odd resonance conditions at a time, the impedance factors in Eqs. (B-8) are not zero. Thus,

$$I1 = I2 \quad \text{(B-9a)}$$

$$I3 = I4\ . \quad \text{(B-9b)}$$

C. DERIVATION OF EQS. (2-17) AND (2-18)

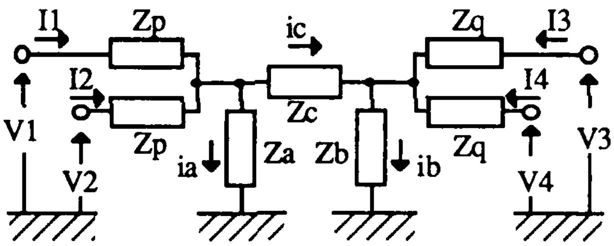


Fig. C-1 Equivalent circuit with V I definitions

Give the voltage and current definition references as in Fig. C-1 to obtain circuit equations :

$$I1 + I2 = ia + ic \quad \text{(C-1a)}$$

$$I3 + I4 = ia - ic \quad \text{(C-1b)}$$

$$Za\, ia = Zb\, ib + Zc\, ic \quad \text{(C-1c)}$$

$$V1 = Zp\, I1 + Za\, ia \quad \text{(C-1d)}$$

$$V2 = Zp\, I2 + Za\, ia \quad \text{(C-1e)}$$

$$V3 = Zq\, I3 + Zb\, ib \quad \text{(C-1f)}$$

$$V4 = Zq\, I4 + Zb\, ib \quad \text{(C-1g)}$$

From Eq. (C-1a) and (C-1b), we get

$$ia + ib = I1 + I2 + I3 + I4 \quad . \tag{C-2}$$

From Eq. (C-1b) and (C-1c), we get

$$Za\, ia - (Zb + Zc)\, ib = - Zc\,(I3 + I4) \quad . \tag{C-3}$$

From Eq. (C-2) and (C-3), ia and ib are obtaind :

$$ia = \frac{Zb+Zc}{Za+Zb+Zc} I1 + \frac{Zb+Zc}{Za+Zb+Zc} I2 + \frac{Zb}{Za+Zb+Zc} I3 + \frac{Zb}{Za+Zb+Zc} I4 \tag{C-4a}$$

$$ib = \frac{Za}{Za+Zb+Zc} I1 + \frac{Za}{Za+Zb+Zc} I2 + \frac{Zc+Za}{Za+Zb+Zc} I3 + \frac{Zc+Za}{Za+Zb+Zc} I4 \tag{C-4b}$$

Inserting Eqs. (C-4) into Eqs. from (C-1d) to (C-1g), system of V-I equations are obtained as follows :

$$V1 = \left(Zp + \frac{Za(Zb+Zc)}{Za+Zb+Zc}\right) I1 + \frac{Za(Zb+Zc)}{Za+Zb+Zc} I2 + \frac{Za\,Zb}{Za+Zb+Zc} I3 + \frac{Za\,Zb}{Za+Zb+Zc} I4 \tag{C-5a}$$

$$V2 = \frac{Za(Zb+Zc)}{Za+Zb+Zc} I1 + \left(Zp + \frac{Za(Zb+Zc)}{Za+Zb+Zc}\right) I2 + \frac{Za\,Zb}{Za+Zb+Zc} I3 + \frac{Za\,Zb}{Za+Zb+Zc} I4 \tag{C-5b}$$

$$V3 = \frac{Za\,Zb}{Za+Zb+Zc} I1 + \frac{Za\,Zb}{Za+Zb+Zc} I2 + \left(Zq + \frac{Zb(Zc+Za)}{Za+Zb+Zc}\right) I3 + \frac{Zb(Zc+Za)}{Za+Zb+Zc} I4 \tag{C-5c}$$

$$V4 = \frac{Za\,Zb}{Za+Zb+Zc} I1 + \frac{Za\,Zb}{Za+Zb+Zc} I2 + \frac{Zb(Zc+Za)}{Za+Zb+Zc} I3 + \left(Zq + \frac{Zb(Zc+Za)}{Za+Zb+Zc}\right) I4 \tag{C-5d}$$

or

$$\begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \end{bmatrix} = \begin{bmatrix} Z11 & Z12 & Z13 & Z13 \\ Z12 & Z11 & Z13 & Z13 \\ Z13 & Z13 & Z33 & Z34 \\ Z13 & Z13 & Z34 & Z33 \end{bmatrix} \begin{bmatrix} I1 \\ I2 \\ I3 \\ I4 \end{bmatrix} \tag{C-5e}$$

where

$$Z11 = Zp + \frac{Za\,(Zb + Zc)}{Za + Zb + Zc} \tag{C-6a}$$

$$Z12 = \frac{Za\,(Zb + Zc)}{Za + Zb + Zc} \tag{C-6b}$$

$$Z13 = \frac{Za\,Zb}{Za + Zb + Zc} \tag{C-6c}$$

$$Z33 = Zq + \frac{Zb\,(Zc + Za)}{Za + Zb + Zc} \tag{C-6d}$$

$$Z34 = \frac{Zb\,(\,Zc + Za\,)}{Za + Zb + Zc} \qquad \text{(C-6e)}$$

Next, we derive Za, Zb,--, Zq as functons of Z parameters. For this, we solve Eqs. (C-6). Divide Eq. (C-6b) by Eq. (C-6c) to obtain

$$\frac{Z12}{Z13} = 1 + \frac{Zc}{Zb}\ . \qquad \text{(C-7)}$$

Divide Eq. (C-6e) by Eq. (C-6c) to obtain

$$\frac{Z34}{Z13} = 1 + \frac{Zc}{Za}\ . \qquad \text{(C-8)}$$

Modify Eq. (C-6c) to get

$$1 + \frac{Zc}{Zb} = Za\,(\,\frac{1}{Z13} - \frac{1}{Zb}\,) \qquad \text{(C-9a)}$$

$$1 + \frac{Zc}{Za} = Zb\,(\,\frac{1}{Z13} - \frac{1}{Za}\,)\ . \qquad \text{(C-9b)}$$

Substitute Eqs. (C-9) into Eq. (C-7) and (C-8) to obtain

$$\frac{Z12}{Z13} = Za\,(\,\frac{1}{Z13} - \frac{1}{Zb}\,) \qquad \text{(C-10a)}$$

$$\frac{Z34}{Z13} = Zb\,(\,\frac{1}{Z13} - \frac{1}{Za}\,) \ \text{or}\ \frac{1}{Zb} = \frac{Z13}{Z34}\,(\,\frac{1}{Z13} - \frac{1}{Za}\,)\ . \qquad \text{(C-10b)}$$

Eliminate Zb in Eq. (C-10a) by substituting Eq. (C-10b) to obtain

$$Za = \frac{Z13^2 - Z12\,Z34}{Z13 - Z34}\ . \qquad \text{(C-11)}$$

Inserting the result of Eq. (C-11) backward, all parameters are evaluated :

$$Zb = \frac{Z13^2 - Z12\,Z34}{Z13 - Z12} \qquad \text{(C-12)}$$

$$Zc = -\,\frac{Z13^2 - Z12\,Z34}{Z13} \qquad \text{(C-13)}$$

$$Zp = Z11 - Z12 \qquad \text{(C-14)}$$

$$Zq = Z33 - Z34 \qquad \text{(C-15)}$$

D. NORMALIZED IMPEDANCE MATRIX

For an N-port network, the impedance matrix is defined as

$$[\,\mathbf{V}\,] = [\,\mathbf{Z}\,]\,[\,\mathbf{I}\,] \qquad \text{(D-1)}$$

where

$$[\,\mathbf{V}\,] = \begin{bmatrix} V1 \\ V2 \\ \vdots \\ VN \end{bmatrix} \qquad [\,\mathbf{I}\,] = \begin{bmatrix} I1 \\ I2 \\ \vdots \\ IN \end{bmatrix}$$

$$[\,\mathbf{Z}\,] = \begin{bmatrix} Z11 & Z12 & - & Z1N \\ Z21 & Z22 & & \vdots \\ \vdots & & \ddots & \vdots \\ \vdots & & & \vdots \\ ZN1 & - & - & ZNN \end{bmatrix}$$

Now, we introduce a normalized voltage vector and a normalized current vector defined as

$$[\,\mathbf{v}\,] = \mathrm{diag}\,[\,\sqrt{Zoi}^{-1}\,]\,[\,\mathbf{V}\,] \qquad \text{(D-2a)}$$

$$[\,\mathbf{i}\,] = \mathrm{diag}\,[\,\sqrt{Zoi}\,]\,[\,\mathbf{I}\,] \qquad \text{(D-2b)}$$

where Zoi is the characteristic impdance at the i-th port.

Substitute Eqs. (D-2) into Eq. (D-1) to obtain

$$\begin{aligned}[\,\mathbf{v}\,] &= \mathrm{diag}\,[\,\sqrt{Zoi}\,]^{-1}\,[\,\mathbf{Z}\,]\ \mathrm{diag}\,[\,\sqrt{Zoi}\,]^{-1}\,[\,\mathbf{i}\,] \\ &= [\,\mathbf{z}\,]\,[\,\mathbf{i}\,] \end{aligned} \qquad \text{(D-3)}$$

where

[z] : normalized impedance matrix with each element as

$$zij = \frac{Zij}{\sqrt{Zoi}\,\sqrt{Zoj}} \qquad i, j = 1, 2, --, N \qquad \text{(D-4)}$$

$$*\ \ \mathrm{diag}\,[\,a_i^{-1}\,] = \mathrm{diag}\,[\,a_i\,]^{-1} \qquad \text{(D-5)}$$

Meanwhile, voltages and currents are decomposed into forward wave component and backward wave component :

$$[\,\mathbf{v}\,] = [\,\mathbf{v}\,]^{+} + [\,\mathbf{v}\,]^{-} \qquad \text{(D-6a)}$$

$$[\,\mathbf{i}\,] = [\,\mathbf{i}\,]^{+} - [\,\mathbf{i}\,]^{-} \qquad \text{(D-6b)}$$

where superscript ($^{+}$) refers to forward quantity and ($^{-}$) refers to backward quantity. S matrix is defined commonly in terms of normalized voltage vectors as

$$[\,\mathbf{v}\,]^{-} = [\,\mathbf{S}\,]\,[\,\mathbf{v}\,]^{+}\ . \qquad \text{(D-7)}$$

Substitute Eqs. (D-6) into Eq. (D-3) and make use of the relations that $[v]^{+} = [i]^{+}$ and $[v]^{-} = [i]^{-}$ to obtain

$$[\,v\,]^{+} = [\,[\,z\,] + [\,U\,]\,]^{-1}[\,[\,z\,] - [\,U\,]\,][\,v\,]^{-}$$

or

$$[\,S\,] = [\,[\,z\,] + [\,U\,]\,]^{-1}[\,[\,z\,] - [\,U\,]\,] \qquad \text{(D-8)}$$

where [U] is a unit matrix. S matrix, i.e., S parameters are calculated regarding to normalized z parameters as follows :

$$[\,[\,z\,] + [\,U\,]^{-1}[\,[\,z\,] - [\,U\,]\,] = \frac{\mathrm{adj}\,\|\,[\,z\,] + [\,U\,]\,\|}{\det \|\,[\,z\,] + [\,U\,]\,\|}[\,[\,z\,] - [\,U\,]\,]$$

$$= \frac{1}{\det\|[z]+[U]\|}\begin{bmatrix} a11 & a12 & a13 & a13 \\ a12 & a11 & a13 & a13 \\ a13 & a13 & a33 & a34 \\ a13 & a13 & a34 & a33 \end{bmatrix}\begin{bmatrix} z11-1 & z12 & z13 & z13 \\ z12 & z11-1 & z13 & z13 \\ z13 & z13 & z33-1 & z34 \\ z13 & z13 & z34 & z33-1 \end{bmatrix}$$

$$= \frac{1}{\det\|[z]+[U]\|}\begin{bmatrix} c11 & c12 & c13 & c13 \\ c12 & c11 & c13 & c13 \\ c13 & c13 & c33 & c34 \\ c13 & c13 & c34 & c33 \end{bmatrix} \qquad \text{(D-10a)}$$

where

$$a11 = (z33-z34+1)\,\{\,(z11+1)\,(z33+z34+1) - 2\,z13^2\,\} \qquad \text{(D-10b)}$$

$$a12 = -\,(z33-z34+1)\,\{\,z12\,(z33+z34+1) - 2\,z13^2\,\} \qquad \text{(D-10c)}$$

$$a13 = -\,z13\,(z11-z12+1)\,(z33-z34+1) \qquad \text{(D-10d)}$$

$$a33 = (z11-z12+1)\,\{\,(z33+1)\,(z11+z12+1) - 2\,z13^2\,\} \qquad \text{(D-10e)}$$

$$a34 = -\,(z11-z12+1)\,\{\,z34\,(z11+z12+1) - 2\,z13^2\,\} \qquad \text{(D-10f)}$$

$$c11 = a11\,(z11-1) + a12\,z12 + 2\,a13\,z13 \qquad \text{(D-10g)}$$

$$c12 = a11\,z12 + a12\,(z11-1) + 2\,a13\,z13 \qquad \text{(D-10h)}$$

$$c13 = (a11+a12)\,z13 + a13\,(z33+z34-1) \qquad \text{(D-10i)}$$

$$c33 = 2\,a13\,z13 + a33\,(z33-1) + a34\,z34 \qquad \text{(D-10j)}$$

$$c34 = 2\,a13\,z13 + a33\,z34 + a34\,(z33-1) \qquad \text{(D-10k)}$$

$$\det\|\,[\,z\,] + [\,U\,]\,\|$$

$$= (z11-z12+1)\,(z33-z34+1)\,\{\,(z11+z12+1)(z33+z34+1) - 4\,z13^2\,\} \qquad \text{(D-10l)}$$

E. DERIVATION OF EQS.(3-10)

For the notational convenience, the subscripts mn are mostly omitted in this section and some abbreviation forms are listed in advance :

$$\beta^{(i)} = \beta mn^{(i)} \qquad i = 1, 2, 3$$

$$A^{(i)} = Amn^{(i)} \qquad i = 1, 2$$

$$B^{(i)} = Bmn^{(i)} \qquad i = 1, 2$$

$$D, E, F, G = Dmn, Emn, Fmn, Gmn$$

$$\Psi, \varphi = \Psi mn, \varphi mn$$

$$\xi_{v^{(i)}} = \xi_{mn,v^{(i)}} \qquad i = 1, 2$$

$$\theta_{v^{(i)}} = \theta_{mn,v^{(i)}} \qquad i = 1, 2$$

$$S_1, S_2, S_3 = \sin \beta^{(1)}h1, \sin \beta^{(2)}h2, \sin \beta^{(3)}t$$

$$C_1, C_2, C_3 = \cos \beta^{(1)}h1, \cos \beta^{(2)}h2, \cos \beta^{(3)}t$$

$$Ct_1, Ct_2, Ct_3 = C_1/S_1, C_2/S_2, C_3/S_3$$

$$z\wedge = j\,\omega\,\mu_o$$

Apply the boundary condition of Eq. (3-7a), i.e.,

$$Et_1 = Et_3$$

to Eq. (3-4a) and Eq. (3-6a) at z = 0, and take inner products over S with the orthogonal functions $\nabla t\,\Psi \times \mathbf{z}$ and $\nabla t\,\varphi$ respectively to obtain

$$\int_s \nabla t\,\Psi \times \mathbf{z} \bullet [\,Et_1 - Et_3\,]\,ds = 0 \qquad \text{at } z = 0$$

$$\int_s \nabla t\,\varphi \bullet [\,Et_1 - Et_3\,]\,ds = 0 \qquad \text{at } z = 0$$

$$-A^{(1)}\,S_1 = D\,S_3 \tag{E-1}$$

$$-B^{(1)}\,(\,\beta^{(1)}/y\wedge_1\,)\,S_1 = G\,(\,\beta^{(3)}/y\wedge_3\,)\,S_3\ . \tag{E-2}$$

Take the inner products over S with $\nabla t\,\Psi$, $\nabla t\,\varphi \times \mathbf{z}$ for the boundary condition of Eq. (3-7c) respectively to obtain

$$\int_s \nabla t\,\Psi \bullet [\,Ht_3 - Ht_1\,]\,ds = \int_{s1} \nabla t\,\Psi \bullet \Sigma\,Pv^{(1)}\,\chi v^{(1)}\,ds \qquad \text{at } z = 0$$

$$\int_s \nabla t\,\varphi \times \mathbf{z} \bullet [\,Ht_3 - Ht_1\,]\,ds = \int_{s1} \nabla t\,\varphi \times \mathbf{z} \bullet \Sigma\,Pv^{(1)}\,\chi v^{(1)} \qquad \text{at } z = 0$$

$$\{ C + D\,C_3 \}\,(\beta^{(3)}/z\hat{}) - A^{(1)}\,(\beta^{(1)}/z\hat{})\,C_1 = \Sigma\, Pv^{(1)} \bullet \xi_{v^{(1)}} \qquad \text{(E-3)}$$

$$F + G\,C_3 - B^{(1)}\,C_1 = \Sigma\, Pv^{(1)} \bullet \theta_{v^{(1)}} \qquad \text{(E-4)}$$

where

$$\xi_{v^{(1)}} = \int_{s1} \chi_{v^{(1)}} \bullet \nabla t \Psi \, ds$$

$$\theta_{v^{(1)}} = \int_{s1} \chi_{v^{(1)}} \bullet \nabla t \varphi \times z \, ds$$

Note that the integral region for $\xi_{v^{(1)}}$, $\theta_{v^{(1)}}$ is reduced to S1.

In order to apply the boundary condition of Eq. (3-7b), we may take the inner product with function χ_μ so as to expect the same expressions as ξ_μ, θ_μ for convenience. The H field function χ_μ, however, is orthogonal to Et field. Hence, we use $\chi_\mu \times z$ instead which is parallel to Et and integration is done only over S1 :

$$\int_{s1} \chi_\mu \times z \bullet Et_1 \, ds = 0 \qquad \text{at } z = 0$$

$$\sum_{mn}^{MN} [\, A^{(1)} \int_{s1} \chi_\mu^{(1)} \times z \bullet \nabla t \Psi \times z \, ds + B^{(1)}(\beta^{(1)}/y\hat{}_1) \int_{s1} \chi_\mu^{(1)} \times z \bullet \nabla t \varphi \, ds \,]\, S_1 = 0$$

or

$$\sum_{mn}^{MN} [\, A^{(1)}\, \xi_\mu^{(1)} - B^{(1)}(\beta^{(1)}/y\hat{}_1)\, \theta_\mu^{(1)} \,]\, S_1 = 0 \,, \qquad \mu = 1, 2, 3, - \qquad \text{(E-5)}$$

where following vector identities are used :

$$\chi \times z \bullet \nabla t \Psi \times z = \chi \bullet \nabla t \Psi$$

$$\chi \times z \bullet \nabla t \varphi = - \chi \bullet \nabla t \varphi \times z$$

Similar process is taken for the boundary conditions at the interface (II) to extract unknown coefficents in Es. (3-5) and (3-6).

Take inner products over S with $\nabla t\, \Psi \times z$ and $\nabla t\, \varphi$ respectively for the boundary condition of Eq. (3-8a) to obtain

$$\int_{s} \nabla t\, \Psi \times z \bullet [\, Et_2 - Et_3 \,]\, ds = 0 \qquad \text{at } z = t$$

$$\int_{s} \nabla t\, \varphi \bullet [\, Et_2 - Et_3 \,]\, ds = 0 \qquad \text{at } z = t$$

$$A^{(2)} S_2 = - C S_3 \tag{E-6}$$

$$B^{(2)} (\beta^{(2)}/y\hat{}_2) S_2 = - F (\beta^{(3)}/y\hat{}_3) S_3 \quad . \tag{E-7}$$

Take the inner products over S with $\nabla t\, \Psi$, $\nabla t\, \Phi \times \mathbf{z}$ for the boundary condition of Eq. (3-8c) respectively to obtain

$$\int_s \nabla t\, \Psi \bullet [\, \mathbf{Ht}_2 - \mathbf{Ht}_3 \,]\, ds = \int_{s2} \nabla t\, \Psi \bullet \Sigma\, P\nu^{(2)}\, \chi\nu^{(2)}\, ds \qquad \text{at } z = t$$

$$\int_s \nabla t\, \Phi \times \mathbf{z} \bullet [\, \mathbf{Ht}_2 - \mathbf{Ht}_3 \,]\, ds = \int_{s2} \nabla t\, \Phi \times \mathbf{z} \bullet \Sigma\, P\nu^{(2)}\, \chi\nu^{(2)} \qquad \text{at } z = t$$

$$A^{(2)} (\beta^{(2)}/z\hat{}) C_2 - \{ C\, C_3 + D \} (\beta^{(1)}/z\hat{}) = \Sigma\, P\nu^{(2)} \bullet \xi_\nu^{(2)} \tag{E-8}$$

$$B^{(2)} C_2 - \{ F\, C_3 + G \} = \Sigma\, P\nu^{(2)} \bullet \theta_\nu^{(2)} \tag{E-9}$$

where

$$\xi_\nu^{(2)} = \int_{s2} \chi\nu^{(2)} \bullet \nabla t \Psi\, ds$$

$$\theta_\nu^{(2)} = \int_{s2} \chi\nu^{(2)} \bullet \nabla t \Phi \times \mathbf{z}\, ds$$

Take the inner product over S2 with $\chi\mu \times \mathbf{z}$ for the boundary condition of Eq. (3-8b) to obtain

$$\int_{s2} \chi\mu \times \mathbf{z} \bullet \mathbf{Et}_2\, ds = 0 \qquad \text{at } z = t$$

$$\sum_{mn}^{MN} [\, A^{(2)} \int_{s2} \chi\mu^{(2)} \times \mathbf{z} \bullet \nabla t \Psi \times \mathbf{z}\, ds + B^{(2)}(\beta^{(2)}/y\hat{}_2) \int_{s2} \chi\mu^{(2)} \times \mathbf{z} \bullet \nabla t \Phi\, ds \,]\, S_2 = 0$$

or
$$\sum_{mn}^{MN} [\, A^{(2)}\, \xi_\mu^{(2)} - B^{(2)}(\beta^{(2)}/y\hat{}_2)\, \theta_\mu^{(2)} \,]\, S_2 = 0 \;, \qquad \mu = 1, 2, 3, - \tag{E-10}$$

Eliminate Cmn, Dmn, Fmn, and Gmn in Eqs. (E-3), (E-4), (E-8), and (E-9) by substitution of Eqs. (E-1), (E-2), (E-6), and (E-7) to obtain

$$-[\beta^{(1)}C_1S_3+\beta^{(3)}S_1C_3]A^{(1)}-\beta^{(3)}S_2A^{(2)} = S_3z\hat{}\, \Sigma\, P\nu^{(1)} \bullet \xi_\nu^{(1)} \tag{E-11a}$$

$$\beta^{(3)}S_1A^{(1)}+[\beta^{(2)}C_2S_3+\beta^{(3)}S_2C_3]A^{(2)} = S_3z\hat{}\, \Sigma\, P\nu^{(2)} \bullet \xi_\nu^{(2)} \tag{E-11b}$$

$$-[\beta^{(3)}C_1S_3+(\varepsilon_3/\varepsilon_1)\beta^{(3)}S_1C_3]B^{(1)}-(\varepsilon_3/\varepsilon_2)\beta^{(2)}S_2B^{(2)} = \beta^{(3)}S_3\Sigma\, P\nu^{(1)} \bullet \theta_\nu^{(1)} \tag{E-11c}$$

$$(\varepsilon_3/\varepsilon_1)\beta^{(1)}S_1B^{(1)}+[\beta^{(3)}C_2S_3+(\varepsilon_3/\varepsilon_2)\beta^{(2)}S_2C_3]B^{(2)} = \beta^{(3)}S_3\Sigma\, P\nu^{(2)} \bullet \theta_\nu^{(2)} \,. \tag{E-11d}$$

Solve Eqs. (E-11) for $Amn^{(1)}$, $Amn^{(2)}$, $Bmn^{(1)}$, $Bmn^{(2)}$ to obtain

$$A^{(1)} = a^{(11)} \sum Pv^{(1)} \cdot \xi v^{(1)} + a^{(12)} \sum Pv^{(2)} \cdot \xi v^{(2)} \qquad \text{(E-12a)}$$

$$A^{(2)} = a^{(21)} \sum Pv^{(1)} \cdot \xi v^{(1)} + a^{(22)} \sum Pv^{(2)} \cdot \xi v^{(2)} \qquad \text{(E-12b)}$$

$$B^{(1)} = b^{(11)} \sum Pv^{(1)} \cdot \theta v^{(1)} + b^{(12)} \sum Pv^{(2)} \cdot \theta v^{(2)} \qquad \text{(E-12c)}$$

$$B^{(2)} = b^{(21)} \sum Pv^{(1)} \cdot \theta v^{(1)} + b^{(22)} \sum Pv^{(2)} \cdot \theta v^{(2)} \qquad \text{(E-12d)}$$

where

$$amn^{(11)} = Ka\ (1/S_1)\ (\beta^{(2)}Ct_2 + \beta^{(3)}Ct_3) \qquad \text{(3-12e)}$$

$$amn^{(12)} = Ka\ (1/(S_1 S_3))\ \beta^{(3)} \qquad \text{(3-12f)}$$

$$amn^{(21)} = -Ka\ (1/(S_2 S_3))\ \beta^{(3)} \qquad \text{(3-12g)}$$

$$amn^{(22)} = -Ka\ (1/S_2)\ (\beta^{(1)}Ct_1 + \beta^{(3)}Ct_3) \qquad \text{(3-12h)}$$

$$Ka = z\hat{}\ / \{\beta^{(3)2} - \beta^{(3)}Ct_3(\beta^{(2)}Ct_2 + \beta^{(1)}Ct_1) - \beta^{(1)}\beta^{(2)}Ct_1Ct_2\}$$

$$bmn^{(11)} = Kb\ (1/S_1)\ (\beta^{(3)}Ct_2 + (\varepsilon_3/\varepsilon_2)\beta^{(2)}Ct_3) \qquad \text{(3-12i)}$$

$$bmn^{(12)} = Kb\ (1/(S_1 S_3))\ (\varepsilon_3/\varepsilon_2)\ \beta^{(2)} \qquad \text{(3-12j)}$$

$$bmn^{(21)} = -Kb\ (1/(S_2 S_3))\ (\varepsilon_3/\varepsilon_1)\ \beta^{(1)} \qquad \text{(3-12k)}$$

$$bmn^{(22)} = -Kb\ (1/S_2)\ (\beta^{(3)}Ct_1 + (\varepsilon_3/\varepsilon_1)\beta^{(1)}Ct_3) \qquad \text{(3-12l)}$$

$$Kb = \beta^{(3)} / [\ (\varepsilon_3/\varepsilon_1)(\varepsilon_3/\varepsilon_2)\beta^{(1)}\beta^{(2)} - \beta^{(3)}Ct_3\{(\varepsilon_3/\varepsilon_1)\beta^{(1)}Ct_2 + (\varepsilon_3/\varepsilon_2)\beta^{(2)}Ct_1\} - \beta^{(3)2}Ct_1Ct_2\ ]$$

Finally, substitute Eqs. (E-12) into Eq. (E-5) and Eq. (E-10) to obtain

$$\sum_{v_1}^{v_1 M} Pv^{(1)}\, U\mu v^{(11)} + \sum_{v_2}^{v_2 M} Pv^{(2)}\, U\mu v^{(12)} = 0 \qquad \mu = 1, 2, 3,\ v_1 M \qquad \text{(E-13a)}$$

$$\sum_{v_1}^{v_1 M} Pv^{(1)}\, U\mu v^{(21)} + \sum_{v_2}^{v_2 M} Pv^{(2)}\, U\mu v^{(22)} = 0 \qquad \mu = 1, 2, 3,\ v_2 M \qquad \text{(E-13b)}$$

where

$$U\mu v^{(ij)} = \sum_{mn}^{MN} S_i [\ a^{(ij)}\, \xi\mu^{(i)}\, \xi v^{(j)} - (\beta^{(i)}/y\hat{}_i)\ b^{(ij)}\, \theta\mu^{(i)}\, \theta v^{(j)}\ ] \qquad \text{(E-13c)}$$

$$i, j = 1, 2$$

Eqs. (E-13) are refered to as a homogeneous system of equations in terms of unknown coefficients $Pv^{(i)}$ ($i = 1,2$), and are expressed in the matrix form as follows :

$$[\,\mathbf{U}\,][\,\mathbf{P}\,] = 0$$

or

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} v=1 & 2 & \cdots & v1M & v=1 & \cdots & v2M \end{array} \\
\begin{array}{c} \mu=1 \\ 2 \\ \vdots \\ v1M \\ \mu=1 \\ \vdots \\ v2M \end{array} &
\left[\begin{array}{cccc|ccc}
U_{11}^{(11)} & U_{12}^{(11)} & \cdots & U_{1\,v1M}^{(11)} & U_{11}^{(12)} & \cdots & \\
U_{21}^{(11)} & U_{22}^{(11)} & & \vdots & \vdots & \ddots & \vdots \\
\vdots & & \ddots & \vdots & \vdots & & \vdots \\
U_{v1M\,1}^{(11)} & \cdots & & U_{v1M\,v1M}^{(11)} & & \cdots & U_{v1M\,v2M}^{(12)} \\
\hline
U_{11}^{(21)} & \cdots & & & U_{11}^{(22)} & \cdots & \\
\vdots & \ddots & & \vdots & \vdots & \ddots & \vdots \\
U_{v2M\,1}^{(21)} & \cdots & & U_{v2M\,v1M}^{(21)} & & \cdots & U_{v2M\,v2M}^{(22)}
\end{array}\right]
\end{array}
\left[\begin{array}{c}
P_1^{(1)} \\ P_2^{(1)} \\ \vdots \\ P_{v1M}^{(1)} \\ \hline P_2^{(2)} \\ \vdots \\ P_{v2M}^{(2)}
\end{array}\right] = 0
\qquad \text{(E-14)}
$$

Integral values of $\xi_{v}^{(i)}$, $\theta_{v}^{(i)}$ are derived as follows. First, we get the scalar potantial vector functions as

$$\nabla t\, \Psi = -P\left(x \frac{m\pi}{a} \sin \frac{m\pi}{a}x \cos \frac{n\pi}{b}y + y \frac{n\pi}{b} \cos \frac{m\pi}{a}x \sin \frac{n\pi}{b}y \right) \qquad \text{(E-15a)}$$

$$\nabla t\, \varphi \times \mathbf{z} = P\left(x \frac{n\pi}{b} \sin \frac{m\pi}{a}x \cos \frac{n\pi}{b}y - y \frac{m\pi}{a} \cos \frac{m\pi}{a}x \sin \frac{n\pi}{b}y \right) \qquad \text{(E-15b)}$$

where

$$P = \sqrt{\frac{\delta m\, \delta n}{a\, b}}\ \frac{1}{kc} \qquad \delta i = \begin{cases} 1 & i = 0 \\ 2 & i \neq 0 \end{cases}$$

$$kc^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

Take integrals of the product of Eqs. (E-15) and Eqs. (3-13), (3-14), we get

$$\xi_{x,mn,rs}^{(1)} = \int_{s1} \chi_{x,rs}^{(1)} \cdot \nabla_t \Psi_{mn}\, ds$$

$$= -P \frac{m\pi}{a} \int_0^a \sin \frac{m\pi}{a} x \sin \frac{r\pi}{a} x\, dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \cos \frac{n\pi}{b} y \sin \frac{s\pi}{w1}\left(y - \frac{b-w1}{2}\right) dy$$

$$= -P \frac{m\pi}{a} \frac{a}{2} \delta rm\, HI(s,n,w1,b) \qquad \text{(E-16)}$$

where

$$\delta ij = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

HI(s,n,w1,b)

$$\begin{cases} = \dfrac{w1}{2} \sin \dfrac{(s-n)\pi}{2} & \text{for } \dfrac{n}{b} = \dfrac{s}{w1} \\ = \dfrac{(s\pi/w1)}{(s\pi/w1)^2 - (n\pi/b)^2}\left[\cos \dfrac{n\pi(b-w1)}{2b} - (-1)^{-s} \cos \dfrac{n\pi(b+w1)}{2b}\right. & \text{for } \dfrac{n}{b} \neq \dfrac{s}{w1} \end{cases} \qquad \text{(E-17)}$$

$$\xi_{y,mn,rs}^{(1)} = \int_{s1} \chi_{y,rs}^{(1)} \cdot \nabla_t \Psi_{mn}\, ds$$

$$= -P \frac{n\pi}{b} \int_0^a \cos \frac{m\pi}{a} x \cos \frac{r\pi}{a} x\, dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \sin \frac{n\pi}{b} y \frac{\cos \frac{s\pi}{w1}\left\{y - \frac{(b-w1)}{2}\right\}}{\sqrt{1 - \left\{\frac{y-b/2}{w1/2}\right\}^2}} dy$$

$$= -P \frac{n\pi}{b} \frac{a}{\delta m} \delta rm\, SI(s,n,w1,b) \qquad \text{(E-18)}$$

where

SI(s,n,w1,b)

$$\begin{cases} = 0 & n+s : \text{even} \\ = \dfrac{\pi w1}{4}\left[(-1)^{\frac{(n-s-1)}{2}} Jo\left(\left|\dfrac{n\pi}{b} - \dfrac{s\pi}{w1}\right| \dfrac{w1}{2}\right) + (-1)^{\frac{(n+s-1)}{2}} Jo\left(\left|\dfrac{n\pi}{b} + \dfrac{s\pi}{w1}\right| \dfrac{w1}{2}\right)\right] & n+s : \text{odd} \end{cases} \qquad \text{(E-19)}$$

Jo : Zero order Bessel function of the first kind

$$\theta_{x,mn,rs}^{(1)} = \int_{s1} \chi_{x,rs}^{(1)} \cdot \nabla_t \varphi_{mn} \times \mathbf{z}\, ds$$

$$= P \frac{n\pi}{b} \int_0^a \sin \frac{m\pi}{a} x \sin \frac{r\pi}{a} x\, dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \cos \frac{n\pi}{b} y \sin \frac{s\pi}{w1}\left(y - \frac{b-w1}{2}\right) dy$$

$$= P \frac{n\pi}{b} \frac{a}{2} \delta rm\, HI(s,n,w1,b) \qquad \text{(E-20)}$$

$$\theta_{y,mn,rs}^{(1)} = \int_{s1} \chi_{y,rs}^{(1)} \cdot \nabla_t \varphi_{mn} \times z \, ds$$

$$= -P \frac{m\pi}{a} \int_0^a \cos \frac{m\pi}{a} x \cos \frac{r\pi}{a} x \, dx \int_{\frac{(b-w1)}{2}}^{\frac{(b+w1)}{2}} \sin \frac{n\pi}{b} y \frac{\cos \frac{s\pi}{w1}\{y - \frac{(b-w1)}{2}\}}{\sqrt{1 - \{\frac{y-b/2}{w1/2}\}^2}} dy$$

$$= -P \frac{m\pi}{a} \frac{a}{\delta m} \delta rm \, SI(s,n,w1,b) \qquad \text{(E-21)}$$

$$\xi_{x,mn,rs}^{(2)} = \int_{s2} \chi_{x,rs}^{(2)} \cdot \nabla_t \Psi_{mn} \, ds$$

$$= -P \frac{m\pi}{a} \int_0^b \cos \frac{n\pi}{b} y \cos \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \sin \frac{m\pi}{a} x \frac{\cos \frac{r\pi}{w2}\{x - \frac{(a-w2)}{2}\}}{\sqrt{1 - \{\frac{x-a/2}{w2/2}\}^2}} dx$$

$$= -P \frac{m\pi}{a} \frac{b}{\delta n} \delta sn \, SI(r,m,w2,a) \qquad \text{(E-22)}$$

$$\xi_{y,mn,rs}^{(2)} = \int_{s2} \chi_{y,rs}^{(2)} \cdot \nabla_t \Psi_{mn} \, ds$$

$$= -P \frac{n\pi}{b} \int_0^b \sin \frac{n\pi}{b} y \sin \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \cos \frac{m\pi}{a} x \sin \frac{r\pi}{w2}(x - \frac{a-w2}{2}) \, dx$$

$$= -P \frac{n\pi}{b} \frac{b}{2} \delta sn \, HI(r,m,w2,a) \qquad \text{(E-23)}$$

$$\theta_{x,mn,rs}^{(2)} = \int_{s2} \chi_{x,rs}^{(2)} \cdot \nabla_t \varphi_{mn} \times z \, ds$$

$$= P \frac{n\pi}{b} \int_0^b \cos \frac{n\pi}{b} y \cos \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \sin \frac{m\pi}{a} x \frac{\cos \frac{r\pi}{w2}\{x - \frac{(a-w2)}{2}\}}{\sqrt{1 - \{\frac{x-a/2}{w2/2}\}^2}} dx$$

$$= P \frac{n\pi}{b} \frac{b}{\delta n} \delta sn \, SI(r,m,w2,a) \qquad \text{(E-24)}$$

$$\theta_{y,mn,rs}^{(2)} = \int_{s2} \chi_{y,rs}^{(2)} \cdot \nabla_t \varphi_{mn} \times z \, ds$$

$$= -P \frac{m\pi}{a} \int_0^b \sin \frac{n\pi}{b} y \sin \frac{s\pi}{b} y \, dy \int_{\frac{(a-w2)}{2}}^{\frac{(a+w2)}{2}} \cos \frac{m\pi}{a} x \sin \frac{r\pi}{w2}(x - \frac{a-w2}{2}) \, dx$$

$$= -P \frac{m\pi}{a} \frac{b}{2} \delta sn \, HI(r,m,w2,a) \qquad \text{(E-25)}$$

Here are summary of $\xi_{V^{(i)}}$, $\theta_{V^{(i)}}$:

$$\xi_{x^{(1)}} = - Pmn \frac{m\pi}{a} \frac{a}{2} \delta rm\, HI(s,n,w1,b) \quad \text{(E-26a)}$$

$$\xi_{y^{(1)}} = - Pmn \frac{n\pi}{b} \frac{a}{\delta m} \delta rm\, SI(s,n,w1,b) \quad \text{(E-26b)}$$

$$\xi_{x^{(2)}} = - Pmn \frac{m\pi}{a} \frac{b}{\delta n} \delta sn\, SI(r,m,w2,a) \quad \text{(E-26c)}$$

$$\xi_{y^{(2)}} = - Pmn \frac{n\pi}{b} \frac{b}{2} \delta sn\, HI(r,m,w2,a) \quad \text{(E-26d)}$$

$$\theta_{x^{(1)}} = Pmn \frac{n\pi}{b} \frac{a}{2} \delta rm\, HI(s,n,w1,b) \quad \text{(E-26e)}$$

$$\theta_{y^{(1)}} = - Pmn \frac{m\pi}{a} \frac{a}{\delta m} \delta rm\, SI(s,n,w1,b) \quad \text{(E-26f)}$$

$$\theta_{x^{(2)}} = Pmn \frac{n\pi}{b} \frac{b}{\delta n} \delta sn\, SI(r,m,w2,a) \quad \text{(E-26g)}$$

$$\theta_{y^{(2)}} = - Pmn \frac{m\pi}{a} \frac{b}{2} \delta sn\, HI(r,m,w2,a) \quad \text{(E-26h)}$$

$$\delta i = \begin{cases} 1 & i = 0 \\ 2 & i \neq 0 \end{cases}$$

$$\delta ij = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

HI(s,n,w1,b)

$$\begin{cases} = \dfrac{w1}{2} \sin \dfrac{(s-n)\pi}{2} & \text{for } \dfrac{n}{b} = \dfrac{s}{w1} \\ = \dfrac{(s\pi/w1)}{(s\pi/w1)^2 - (n\pi/b)^2}\left[\cos \dfrac{n\pi(b-w1)}{2b} - (-1)^{-s} \cos \dfrac{n\pi(b+w1)}{2b}\right. & \text{for } \dfrac{n}{b} \neq \dfrac{s}{w1} \end{cases}$$

SI(s,n,w1,b)

$$\begin{cases} = 0 & n + s : \text{even} \\ = \dfrac{\pi w1}{4} \left[(-1)^{\frac{(n-s-1)}{2}} Jo\left(\left|\dfrac{n\pi}{b} - \dfrac{s\pi}{w1}\right)\dfrac{w1}{2}\right|\right) + (-1)^{\frac{(n+s-1)}{2}} Jo\left(\left|\dfrac{n\pi}{b} + \dfrac{s\pi}{w1}\right)\dfrac{w1}{2}\right|\right) \right] & n + s : \text{odd} \end{cases}$$

F. CHOICE OF rs COMBINATION

The numbers for r and s and the choice of x or y component for ξ_μ and θ_μ are assigned according to the number ν in Eq. (E-14). An example of rs combination is presented here ; actually, the results in chapter 4 are calculated by this combination. Table F-1 shows the example. In Table F-1, as M-M wall case with $\nu = 1$ (odd number), for instance, $\chi_y^{(1)}$ is chosen at interface (I) for ξ_μ and θ_μ calculations so that the first combination of r' = 0 and s' = 0 is assigned. Next r' s' combination for $\nu = 3$ is that r' = 1 and s' = 0. With ν for even number, χ_x and is chosen.

Table F-1 rs-table

at Interface(I)

Wall (A)	Wall (B)	ν=even χ_x (Jy)	ν=odd χ_y (Jx)	
M	M	r'=0,1,2,-- s'=1	r'=0,1,2,-- s'=0	r=2r'+1 s=2s'
E	M	r'=1,2,3,-- s'=1	r'=0,1,2,-- * s'=0	r=2r' s=2s'
M	E	r'=0,1,2,-- s'=0	r'=0,1,2,-- s'=0	r=2r'+1 s=2s'+1

at Interface(II)

Wall		v=odd χ_x (Jy)	v=even χ_y (Jx)	
(A)	(B)			
M	M	r'=0 s'=0,1,2,--	r'=1 s'=0,1,2,--	r=2r' s=2s'+1
E	M	r'=0 s'=0,1,2,--	r'=0 s'=0,1,2,--	r=2r'+1 s=2s'+1
M	E	r'=0 s'=0,1,2,-- *	r'=1 s'=1,2,3,--	r=2r' s=2s'

* The foundamental resonance mode is described as r'=1(interface(I)) or s'=1(interface(II)) instead of the first combination with r'=0 or s'=0

Note that transverse distribution of each current, i.e., longitudinal or transverse current, is represented only with one term in Table F-1; transverse distribution may be well expressed only with one term while many terms are needed for the expansion of londitudinal distribution because of discontinuiteis.

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PROGRAM LIST

```
      PROGRAM CROSS2
*
*     ==========================================
*     Subroutines in IMSL are required in LINKing
*     ==========================================
*
      Real      Ea(180)
      Common    /size/a,b,w1,w2,h1,h2,t
      Common    /Para/Freq,Er1,Er2,Er3
      Common    /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
      Common    /Mat /Ea
*
      External RSNTR1
*
      OPEN(FILE='CROSS2.DAT',UNIT=7,STATUS='OLD')
*
*      write(*,901)
  901 format(
     -  '         Cross-Striplines                 1986-0227,1986-0404'
     - /'                                                by Tom Uwano'/
     - /'    y                                                     '
     - /'                w2                                        '
     - /'    b  ---------------------                          z  '
     - /'       l        i  i        l                              '
     - /'       l        i  i        l     -------------------- h2'
     - /'       l        i  i        l     l   Er2      (II)  l   '
     - /'       l--------i--i--------l     l       w2         l   '
     - /' b/2   l        i  i        lw1 (III)l-------***--------l t '
     - /'       l--------i--i--------l    Er3 l=================l 0 '
     - /'       l        i  i        l     l                  l   '
     - /'       l        i  i        l     l   Er1      (I)   l   '
     - /'       l        i  i        l     -------------------- -h1'
     - /'     z @---------------------                            '
     - /'       0         a/2        a   x                         '
     - /'                                      Frequency [MHz]   '/)
*
*
      write(*,*)'Input 0(MMwall),1(EMwall,x=a/2) or 2(MEwall,y=b/2)'
      read(*,*) MEwall
      if(MEwall.EQ.0)
     -  write(*,*)'Input 0(combination),1(region(I)) or 2(region(II))'
      if(MEwall.EQ.1) write(*,*)'Input 0(combination) or 1(region(I))'
      if(MEwall.EQ.2) write(*,*)'Input 0(combination) or 2(region(II))'
      read(*,*) Nreso
      if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
           write(*,*)'Input approximate a in [mm]'
           read(*,*)a
           write(*,*)'Input b(fixed) in [mm]'
           read(*,*) b
        else
           write(*,*)'Input approximate b in [mm]'
```

```
          read(*,*) b
          write(*,*)'Input a(fixed) in [mm]'
          read(*,*)a
        endif
      write(*,*)'Input h1,t,h2 in [mm]'
      read(*,*) h1,t,h2
      write(*,*)'Input w1,w2 in [mm]'
      read(*,*) w1,w2
      write(*,*)'Input frequency in [MHz]'
      read(*,*) freq
      write(*,*)'Input Er1,Er2,Er3'
      read(*,*) Er1,Er2,Er3
*
      a = a*1.e-3
      b = b*1.e-3
      h1=h1*1.e-3
      t = t*1.e-3
      h2=h2*1.e-3
      w1=w1*1.e-3
      w2=w2*1.e-3
      freq=freq*1.e-3
      if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
          ax=a
        else
          ax=b
        endif
*
      write(*,*)'Input m,n max'
      read(*,*) Nmax
      write(*,*)'Matrix size (2N x 2N). Input N'
      read(*,*)NHmat
      Np=2*NHmat-1
*
      q=freq*1.e3
      ME=MEwall
      write(7,902)a,b,h1,t,h2,w1,w2,q,Er1,Er2,Er3,Nmax,NHmat,ME,Nreso
  902 format('a =',E13.6/'b =',E13.6/'h1=',E13.6
     -      /'t =',E13.6/'h2=',E13.6/'w1=',E13.6/'w2=',E13.6
     -      /'freq=',E13.6,' MHz'/'Er1,Er2,Er3=',3F7.2
     -      /'m,n max =',I5
     -      /'Matrix 2N x 2N, N=',I3/'MEwall=',I3/'Nreso =',I3)
      icount=0
      coef1=1.
      Call ZERO2(RSNTR1,ax,ier)
      if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
          write(*,*)' final a in [mm] =',ax*1.e3,ier
          write(7,*)' final a in [mm] =',ax*1.e3,ier
        else
          write(*,*)' final b in [mm] =',ax*1.e3,ier
          write(7,*)' final b in [mm] =',ax*1.e3,ier
        endif
*
      write(*,'(4E16.7)') 1.,(Ea(i),i=1,Np)
      write(7,'(4E16.7)') 1.,(Ea(i),i=1,Np)
```

```
      stop
      end
*
*
      FUNCTION RSNTR1(ax)
*
*    Cross-Striplines                                1986-0226,1986-0401
*                                                    by Tom Uwano
*         y
*                     w2
*         b   ----------------------                              z
*             l        i  i        l
*             l        i  i        l        ---------------------- h2
*             l        i  i        l        l   Er2        (II)  l
*             l--------i--i--------l        l       w2           l
*       b/2   l        i  i        lw1  (III)l---------***---------l t
*             l--------i--i--------l    Er3 l====================l 0
*             l        i  i        l        l                    l
*             l        i  i        l        l   Er1        (I)   l
*             l        i  i        l        ---------------------- -h1
*          z  @---------------------
*             0       a/2          a    x
*                                                    Frequency [GHz]
*
      Integer*4    MNmax,mn
      Real         mu0,KK0,Kk1,Kk2,Kk3
      Real         U(2,2),El(32400),Ed(180),wk(180),Es1(8100),Es2(8100)
      Real         Ep(32041),Ea(180)
      Complex      JJ
      Complex      zhat,yhat1,yhat2
      Complex      Btmn1,Btmn2,Btmn3
      Complex      a11,a12,a21,a22,b11,b12,b21,b22
      Complex      SI1,SI2,SI3,CO1,CO2,CO3,TA1,TA2,TA3
      Complex      dela,delb,zdela,b3delb
      Complex      x,FARG,CMCOS,CMSIN
*
      Common       /size/a,b,w1,w2,h1,h2,t
      Common       /Para/Freq,Er1,Er2,Er3
      Common       /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
      Common       /Mat /Ea
*
      Parameter    (JJ=(0.,1.),pai=3.1415926)
      Parameter    (epsi0=8.855e-12,mu0=pai*4.e-7)
*
*------  Function definition  ----------
*
      FARG(X)=REAL(X)+JJ*SIGN(1.,AIMAG(X))*AMIN1(70.,ABS(AIMAG(X)))
      CMCOS(X)=CCOS(FARG(X))
      CMSIN(X)=CSIN(FARG(X))
*
*--------------------------------------
*
      if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
         a=ax
```

```
        else
          b=ax
        endif
       Nmat=2*NHmat
       Np=Nmat-1
       NHp=NHmat-1
*
       Er31=Er3/Er1
       Er32=Er3/Er2
       Omega=2.*pai*Freq*1.e9
       Kk0=Omega*Omega*mu0*epsi0
       Kk1=Er1*Kk0
       Kk2=Er2*Kk0
       Kk3=Er3*Kk0
       zhat=JJ*Omega*mu0
       yhat1=JJ*Omega*epsi0*Er1
       yhat2=JJ*Omega*epsi0*Er2
*
*       ------- Matrix Size   N x N  (N= 2 x Mumax)
       Mumax=NHmat
       Numax=Mumax
*      -------------------------------------------
       Nmax=Nmax
       MNmax=Nmax*Nmax
*
       do 201 mu=1,Mumax
       do 201 nu=1,Numax
*
       Call RSTBL(ir1,is1,ixy1,mu,1)
       Call RSTBL(ir2,is2,ixy2,mu,2)
       Call RSTBL(jr1,js1,jxy1,nu,1)
       Call RSTBL(jr2,js2,jxy2,nu,2)
*
* --------------------Begining of Summation in terms of M,N---
*
       U(1,1)=0.
       U(1,2)=0.
       U(2,1)=0.
       U(2,2)=0.
*
       do 202 mn=1,MNmax
          Call MNTBL(m,n,mn)
*
       if(ir1.NE.m.AND.jr1.NE.m.AND.is2.NE.n.AND.js2.NE.n) go to 1
*
       GGmn=(Float(m)*pai/a)**2+(Float(n)*pai/b)**2
       Btmn1=CSQRT(CMPLX(Kk1-GGmn))
       Btmn2=CSQRT(CMPLX(Kk2-GGmn))
       Btmn3=CSQRT(CMPLX(Kk3-GGmn))
       CO1=CMCOS(Btmn1*h1)
       CO2=CMCOS(Btmn2*h2)
       CO3=CMCOS(Btmn3*t )
       SI1=CMSIN(Btmn1*h1)
       SI2=CMSIN(Btmn2*h2)
```

```
      SI3=CMSIN(Btmn3*t )
      TA1=SI1/CO1
      TA2=SI2/CO2
      TA3=SI3/CO3
      dela=Btmn3*Btmn3*TA1*TA2*TA3-Btmn3*(Btmn2*TA1+Btmn1*TA2)
     -                             -Btmn1*Btmn2*TA3
      delb=Btmn1*Btmn2*Er31*Er32*TA1*TA2*TA3
     -     -Btmn3*(Btmn1*Er31*TA1+Btmn2*Er32*TA2)-Btmn3*Btmn3*TA3
      zdela=zhat/dela
      a11= zdela*(Btmn2*TA3+Btmn3*TA2)
      a12= (1./CO3)*zdela*Btmn3*TA2
      a21=-(1./CO3)*zdela*Btmn3*TA1
      a22=-zdela*(Btmn1*TA3+Btmn3*TA1)
      b3delb=Btmn3/delb
      b11= b3delb*(Btmn3*TA3+Btmn2*Er32*TA2)
      b12= (1./CO3)*b3delb*Btmn2*Er32*TA2
      b21=-(1./CO3)*b3delb*Btmn1*Er31*TA1
      b22=-b3delb*(Btmn3*TA3+Btmn1*Er31*TA1)
*
      Call  GSITH1(Gmu1,Tmu1,ixy1,m,n,ir1,is1)
      Call  GSITH2(Gmu2,Tmu2,ixy2,m,n,ir2,is2)
      Call  GSITH1(Gnu1,Tnu1,jxy1,m,n,jr1,js1)
      Call  GSITH2(Gnu2,Tnu2,jxy2,m,n,jr2,js2)
*
      U11=AIMAG( TA1*Gmu1*a11*Gnu1-TA1*Btmn1/yhat1*Tmu1*b11*Tnu1)
      U12=AIMAG( TA1*Gmu1*a12*Gnu2-TA1*Btmn1/yhat1*Tmu1*b12*Tnu2)
      U21=AIMAG( TA2*Gmu2*a21*Gnu1-TA2*Btmn2/yhat2*Tmu2*b21*Tnu1)
      U22=AIMAG( TA2*Gmu2*a22*Gnu2-TA2*Btmn2/yhat2*Tmu2*b22*Tnu2)
*
      U(1,1)=U(1,1)+U11
      U(1,2)=U(1,2)+U12
      U(2,1)=U(2,1)+U21
      U(2,2)=U(2,2)+U22
*
    1 continue
*
  202 continue
*
* ------------------------- End of Summation -------
*
      do 203 i=1,2
      do 203 j=1,2
      Call MXTBL(k,l,i,j,nu,mu,Numax)
      kl=k+Nmat*(l-1)
      El(kl)=U(i,j)
  203 continue
*
  201 continue
*
*  -----------process to avoid overflow in the determinant
*
      icount=icount+1
      if(icount.EQ.1) then
          sum=0.
```

```
          do 205 i=1,Nmat
          kl=(i-1)*Nmat+i
 205      sum=sum+ALOG(ABS(El(kl)))
          coefl=EXP((10.-sum)/FLOAT(Nmat))
        else
        endif
*
      do 206 i=1,Nmat*Nmat
 206  El(i)=coefl*El(i)
*
*  ------------------------------------- end of process
*
      do 204 i=1,NHmat
      do 204 j=1,NHmat
      ij=i+NHmat*(j-1)
      kl1=i+Nmat*(j-1)
      kl2=i+NHmat+Nmat*(j+NHmat-1)
      Es1(ij)=El(kl1)
      Es2(ij)=El(kl2)
 204  continue
*
*  -------------------------Finding Pnu--
*
      if(Nreso.EQ.0) then
        do 207 i=1,Np
 207    Ea(i)=-El(i)
        do 208 i=1,Np*Np
        kl=Nmat+i+(i-1)/Np
 208    Ep(i)=El(kl)
        Call LINV3F(Ep,Ea,2,Np,Np,d1,d2,wk,ier)
        endif
      if(Nreso.EQ.1.AND.NHmat.NE.1) Ea(1)=-Es1(1)/Es1(NHmat+1)
      if(Nreso.EQ.2.AND.NHmat.NE.1) Ea(1)=-Es2(1)/Es2(NHmat+1)
*
*  ---------------------------------------
*
      Call LINV3F(Es1,Ed,4,NHmat,NHmat,d1,d2,wk,ier)
      det1=d1*2.**d2
      Call LINV3F(Es2,Ed,4,NHmat,NHmat,d1,d2,wk,ier)
      det2=d1*2.**d2
      Call LINV3F(El,Ed,4,Nmat,Nmat,d1,d2,wk,ier)
      dett=d1*2.**d2
      if(Nreso.EQ.1) RSNTR1=det1
      if(Nreso.EQ.2) RSNTR1=det2
      if(Nreso.EQ.0) RSNTR1=dett
      if((MEwall.EQ.0.AND.Nreso.NE.2).OR.MEwall.EQ.1) then
           write(*,*)a*1.e3,dett,det1,det2
           write(7,*)a*1.e3,dett,det1,det2
        else
           write(*,*)b*1.e3,dett,det1,det2
           write(7,*)b*1.e3,dett,det1,det2
        endif
*
      return
```

```
      END
*
* ------------------------- END of RSNTR1
*
* ------------------------- Begining of Gsi ans Theta calculation
*
      SUBROUTINE GSITH1(G1,T1,ixy,im,in,ir,is)
*
*    Gsai and Theta on the interface btwn (I) and (III)
*                                               1986-0226,1986-0402
      Real      m,n
      Common    /size/a,b,w1,w2
      Parameter  (pai=3.1415926)
      DL(I)=FLOAT(MIN(I+1,2))
      DLIJ(I,J)=FLOAT(1/(IABS(I-J)+1))
*
      m=im
      n=in
      r=ir
      s=is
*
      if(ixy.EQ.1) go to 1
      if(ixy.EQ.2) go to 2
*
*---- x and interface (I)
*
    1 continue
      if(ir.EQ.im) then
          pma=m*pai/a
          pnb=n*pai/b
          GGmn=pma*pma+pnb*pnb
          Gmn=SQRT(GGmn)
          Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
          DD=DLIJ(ir,im)
          Fsn=FI1(is,in,w1,b)
          G1=-Pmn*pma*(a/2.)*DD*Fsn
          T1= Pmn*pnb*(a/2.)*DD*Fsn
        else
          G1=0.
          T1=0.
        endif
      return
*
*---- y and interface (I)
*
    2 continue
      if(ir.EQ.im) then
          pma=m*pai/a
          pnb=n*pai/b
          GGmn=pma*pma+pnb*pnb
          Gmn=SQRT(GGmn)
          Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
          D1=DL(im)
          DD=DLIJ(im,ir)
```

```
        Fsn=FI2(is,in,w1,b)
        G1=-Pmn*pnb*(a/D1)*DD*Fsn
        T1=-Pmn*pma*(a/D1)*DD*Fsn
      else
        G1=0.
        T1=0.
      endif
      return
      end
*
*
      SUBROUTINE GSITH2(G2,T2,ixy,im,in,ir,is)
*
*    Gsai and Theta on the interface btwn (II) and (III)
*                                                 1986-0226,1986-0402
      Real      m,n
      Common    /size/a,b,w1,w2
      Parameter  (pai=3.1415926)
      DL(I)=FLOAT(MIN(I+1,2))
      DLIJ(I,J)=FLOAT(1/(IABS(I-J)+1))
*
      m=im
      n=in
      r=ir
      s=is
*
      if(ixy.EQ.1) go to 1
      if(ixy.EQ.2) go to 2
*
*---- x and interface (II)
*
    1 continue
      if(is.EQ.in) then
          pma=m*pai/a
          pnb=n*pai/b
          GGmn=pma*pma+pnb*pnb
          Gmn=SQRT(GGmn)
          Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
          D1=DL(in)
          DD=DLIJ(is,in)
          Fsn=FI2(ir,im,w2,a)
          G2=-Pmn*pma*(b/D1)*DD*Fsn
          T2= Pmn*pnb*(b/D1)*DD*Fsn
      else
          G2=0.
          T2=0.
      endif
      return
*
*---- y and interface (II)
*
    2 continue
      if(is.EQ.in) then
          pma=m*pai/a
```

```
          pnb=n*pai/b
          GGmn=pma*pma+pnb*pnb
          Gmn=SQRT(GGmn)
          Pmn=SQRT(DL(im)*DL(in)/(a*b))/Gmn
          DD=DLIJ(is,in)
          Fsn=FI1(ir,im,w2,a)
          G2=-Pmn*pnb*(b/2.)*DD*Fsn
          T2=-Pmn*pma*(b/2.)*DD*Fsn
        else
          G2=0.
          T2=0.
        endif
      return
      end
*
* ------------------------------------- end of Gsi and Theta
*
*
      FUNCTION FI1(is,in,w,b)
*
      Real n
      pai=3.1415926
*
      s=is
      n=in
      psw=s*pai/w
      pnb=n*pai/b
      if(pnb.EQ.psw) then
          FI1=-w/2.*SIN((n-s)*pai/2.)
        else
          h= psw/(pnb*pnb-psw*psw)
          c=COS(pnb*(b-w)/2.)-(-1.)**is*COS(pnb*(b+w)/2.)
          FI1=-h*c
        endif
      return
      end
*
      FUNCTION FI2(is,in,w,b)
*
      Real n
      pai=3.1415926
*
      s=is
      n=in
      if(MOD((in+is),2).EQ.0) then
          FI2=0.
        else
          arg1=ABS(n*w*pai/(2.*b)+s*pai/2.)
          arg2=ABS(n*w*pai/(2.*b)-s*pai/2.)
          h=BSJ0(arg1)*(-1.)**((in+is-1)/2)
          c=BSJ0(arg2)*(-1.)**((in-is-1)/2)
          FI2=w*pai/4.*(h+c)
        endif
      return
```

```
      end
*
*
      SUBROUTINE MXTBL(k,l,i,j,Nu,Mu,Nmax)
      k=Mu+Nmax*(i-1)
      l=Nu+Nmax*(j-1)
      return
      end
*
*
      SUBROUTINE RSTBL(ir,is,infoxy,NNu,i)
      Common   /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
*
      if(i.EQ.1) go to 1
      if(i.EQ.2) go to 2
*
    1 continue
*   --------------------------Interface (I)-------------
      if(MOD(NNu,2).EQ.0) then
*                               r,s for the Xx(1)    NNu:even
         infoxy=1
         n=NNu/2
         iir=n-1
         if(MEwall.EQ.1) iir=iir+1
         iis=1
         if(MEwall.EQ.2) iis=0
         if(MEwall.EQ.0) then
           ir=2*iir+1
           is=2*iis
           endif
         if(MEwall.EQ.1) then
           ir=2*iir
           is=2*iis
           endif
         if(MEwall.EQ.2) then
           ir=2*iir+1
           is=2*iis+1
           endif
       else
*                               r,s for the Xy(1)    NNu:odd
         infoxy=2
         n=(NNu+1)/2
         iir=n-1
         iis=0
         if(MEwall.EQ.0) then
           ir=2*iir+1
           is=2*iis
           endif
         if(MEwall.EQ.1) then
           irdum=iir
           if(irdum.EQ.0) iir=1
           if(irdum.EQ.1) iir=0
           ir=2*iir
           is=2*iis
```

```
          endif
        if(MEwall.EQ.2) then
          ir=2*iir+1
          is=2*iis+1
          endif
      endif
      return
*
    2 continue
*      -----------------------Interface (II)-------------
      if(MOD(NNu,2).EQ.0) then
*                                 r,s for the Xy(2)    NNu:even
         infoxy=2
         n=NNu/2
         iis=n-1
         if(MEwall.EQ.2) iis=iis+1
         iir=1
         if(MEwall.EQ.1) iir=0
         if(MEwall.EQ.0) then
           ir=2*iir
           is=2*iis+1
           endif
         if(MEwall.EQ.1) then
           ir=2*iir+1
           is=2*iis+1
           endif
         if(MEwall.EQ.2) then
           ir=2*iir
           is=2*iis
           endif
       else
*                                 r,s for the Xx(2)    NNu:odd
         infoxy=1
         n=(NNu+1)/2
         iis=n-1
         iir=0
         if(MEwall.EQ.0) then
           ir=2*iir
           is=2*iis+1
           endif
         if(MEwall.EQ.1) then
           ir=2*iir+1
           is=2*iis+1
           endif
         if(MEwall.EQ.2) then
           isdum=iis
           if(isdum.EQ.0) iis=1
           if(isdum.EQ.1) iis=0
           ir=2*iir
           is=2*iis
           endif
      endif
      return
      end
```

```
*
*
      SUBROUTINE MNTBL(m,n,MN)
      Common    /Tran/MEwall,Nreso,NHmat,Nmax,icount,coef1
*
*       M - M   Wall    case
      Integer*4 MN
      n=(MN-1)/Nmax
      m=MN-Nmax*n-1
      if(MEwall.EQ.0) then
        m=2*m+1
        n=2*n+1
        endif
      if(MEwall.EQ.1) then
        m=2*m
        n=2*n+1
        endif
      if(MEwall.EQ.2) then
        m=2*m+1
        n=2*n
        endif
      return
      end
*
*
*
      FUNCTION BSJ0(x)
*                         Bessel function       1986-0226
      ax=ABS(x)
      if(ax.GT.3.) go to 1
      x2=x*x/9.
      x4=x2*x2
      x6=x2*x4
      x8=x4*x4
      x10=x2*x8
      x12=x6*x6
      BSJ0=1.-2.2499997d0*x2+1.2656208d0*x4-0.3163866d0*x6
     -       +0.0444479d0*x8-0.0039444d0*x10+0.00021d0*x12
      return
    1 continue
      x1=3./ax
      x2=x1*x1
      x3=x1*x2
      x4=x1*x3
      x5=x1*x4
      x6=x1*x5
      f= 0.79788456d0-0.00000077d0*x1-0.0055274d0*x2-0.00009512d0*x3
     -  +0.00137237d0*x4-0.00072805d0*x5+0.00014476d0*x6
      t=ax-0.78539816d0-0.04166397d0*x1-0.00003954d0*x2+0.00262573d0*x3
     -    -0.00054125d0*x4-0.00029333d0*x5+0.00013558d0*x6
      BSJ0=f*cos(t)/SQRT(ax)
      return
      end
*
```

```
*
*
      SUBROUTINE ZERO2(Func,x,ier)
*                                                   1986-0321,1986-0502
*                                                   by Tom Uwano
*      ier=1 :  no root
*      ier=2 :  dosen't converge
*      5% step toward +
*
      Implicit Real*8(a-h,o-z)
      Real x,Func,ylast
      ier=0
      x1=x
      step=0.05*DABS(x1)
      y1=Func(SNGL(x1))
      ylast=SNGL(y1)
*
      if(ylast.EQ.0.) then
        x=x1
        return
        endif
      x3=x1+step
      y3=Func(SNGL(x3))
      ylast=SNGL(y3)
      if(ylast.EQ.0.) then
        x=x3
        return
        endif
*
      if(y1*y3.LT.0.) then
        x2=x1+(x3-x1)*y1/(y1-y3)
        y2=Func(SNGL(x2))
        xlast=x2
        ylast=SNGL(y2)
        if(ylast.EQ.0.) then
          x=xlast
          return
          endif
      else
        xnew=x1+(x3-x1)*y1/(y1-y3)
        del=y3-y1
        if(del*y1.LT.0.) then
*                           forward step
        x2=x3
        y2=y3
        x3=x3+(x3-x1)
        if(xnew.LT.x3) x3=xnew+0.01*DABS(xnew)
        y3=Func(SNGL(x3))
        xlast=x3
        ylast=SNGL(y3)
        if(y2*y3.LE.0.) go to 100
        do 201 i=1,5
        xnew=x3+(x3-x1)
        x1=x2
```

```
        x2=x3
        x3=xnew
        y1=y2
        y2=y3
        y3=Func(SNGL(x3))
        xlast=x3
        ylast=SNGL(y3)
        if(y2*y3.LE.0.) go to 100
  201   continue
        ier=1
        x=x3
        return
        else
*                                 backward step
        x2=x1
        y2=y1
        x1=x1-(x3-x1)
        if(xnew.GT.x1) x1=xnew-0.01*DABS(xnew)
        y1=Func(SNGL(x1))
        xlast=x1
        ylast=SNGL(y1)
        if(y2*y1.LE.0.) go to 100
        do 202 i=1,5
        xnew=x1-(x3-x1)
        x3=x2
        x2=x1
        x1=xnew
        y3=y2
        y2=y1
        y1=Func(SNGL(x1))
        xlast=x1
        ylast=SNGL(y1)
        if(y2*y1.LE.0.) go to 100
  202   continue
        ier=1
        x=x1
        return
        endif
      endif
*
  100 continue
      if(ylast.EQ.0.) then
        x=xlast
        return
        endif
*
      do 203 i=1,10
      if(i.NE.1) then
        y2=Func(SNGL(x2))
        xlast=x2
        ylast=SNGL(y2)
        if(ylast.EQ.0.) then
          x=xlast
          return
```

```
        endif
      endif
    Call ABCNEW(xnew,x1,x2,x3,y1,y2,y3)
    if(y1*y2.GT.0.) then
      x1=x2
      y1=y2
    else
      x3=x2
      y3=y2
    endif
    x2=xnew
    epsi=1.d-5
    dela=DABS((x2-xlast)/x2)
    if(dela.LT.epsi) then
      x=x2
      return
    endif
203 continue
    ier=2
    x=x2
    return
    end
*
*
*
    SUBROUTINE ABCNEW(xnew,x1,x2,x3,y1,y2,y3)
    Implicit Real*8(a-z)
    Call ABC(a,b,c,x1,x2,x3,y1,y2,y3)
    bb=b
    if(bb.EQ.0.) bb=1.d-30
    if(DABS(a/bb).LT.1.e-10) then
      xnew=-c/b
    else
      ac=b*b-4.*a*c
      anx1=(-b-DSQRT(ac))/(2.*a)
      anx2=(-b+DSQRT(ac))/(2.*a)
      xnew=anx1
      if(anx2.GE.x1.AND.anx2.LE.x3) xnew=anx2
    endif
    return
    end

    SUBROUTINE ABC(a,b,c,x1,x2,x3,y1,y2,y3)
    Implicit Real*8(a-z)
    a11=-1./((x2-x1)*(x1-x3))
    a12=-1./((x2-x1)*(x3-x2))
    a13=-1./((x3-x2)*(x1-x3))
    a21=-a11*(x3+x2)
    a22=-a12*(x1+x3)
    a23=-a13*(x2+x1)
    a31=a11*x2*x3
    a32=a12*x1*x3
    a33=a13*x1*x2
    a=a11*y1+a12*y2+a13*y3
```

```
      b=a21*y1+a22*y2+a23*y3
      c=a31*y1+a32*y2+a33*y3
      return
      end
*
```

END
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DTIC